Objectives

- To discuss the concept of the center of gravity, center of mass, and centroids (centers of area).
- To show how to determine the location of the center of gravity and centroid for a system of particles and a body of arbitrary shape.

Center of Gravity

The center of gravity G is a point which locates the resultant weight of a system of particles.

The weights of the particles is considered to be a parallel force system. The system of weights can be replaced by a single weight acting at the Center of Gravity.

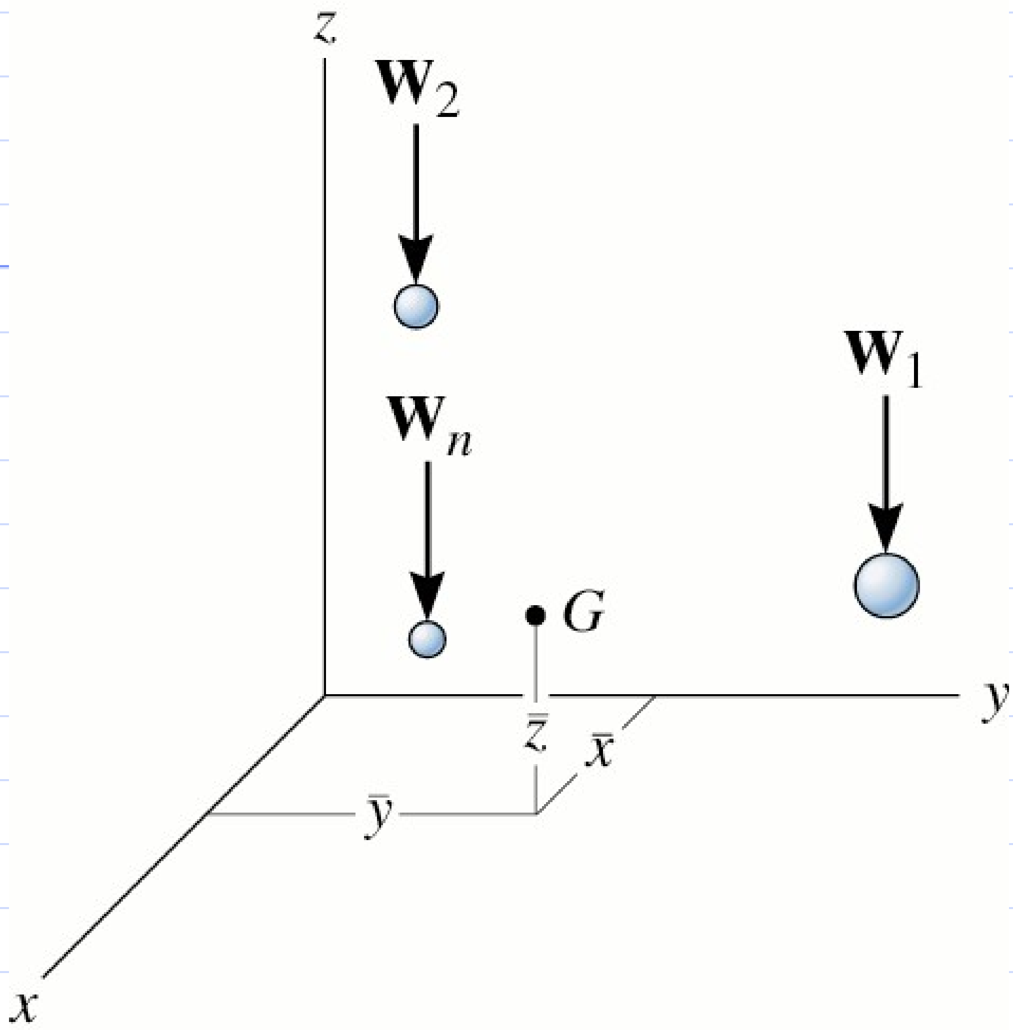


Figure 09.01(a)

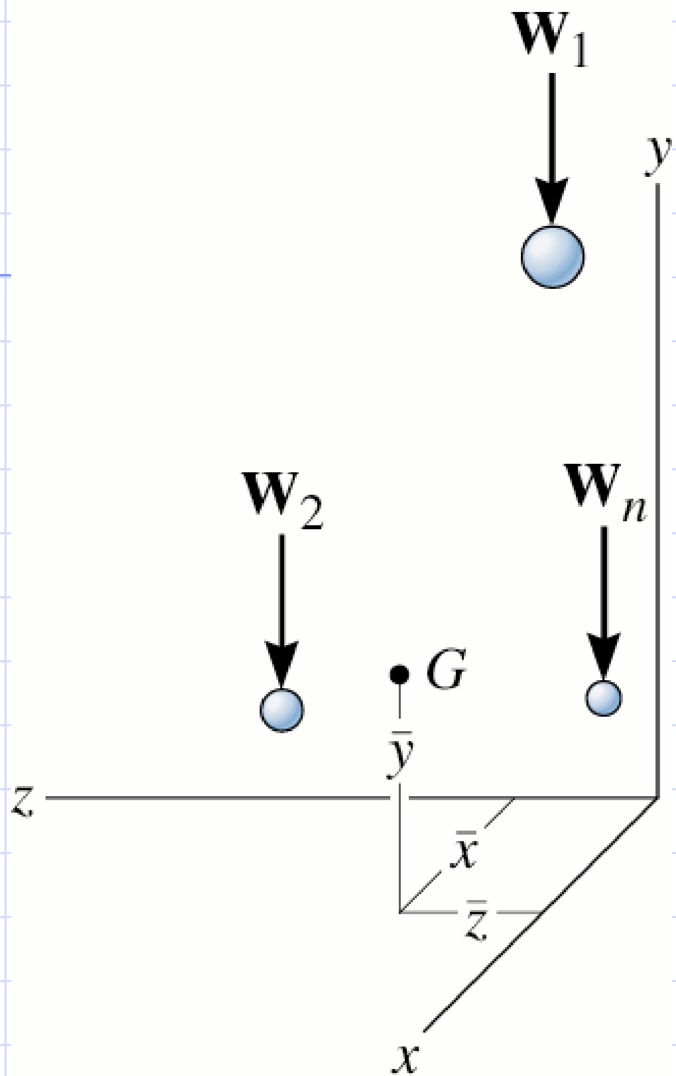


Figure 09.01(b)

$$W_R = \sum_{i=1}^{n} W_i$$

Total Weight

x location:

$$\bar{x}_R W_R = \tilde{x}_1 W_1 + \tilde{x}_2 W_2 + \tilde{x}_3 W_3 + \quad \tilde{x}_n W_n$$

y location:

$$\bar{y}_R W_R = \tilde{y}_1 W_1 + \tilde{y}_2 W_2 + \tilde{y}_3 W_3 + \quad \tilde{y}_n W_n$$

z location:

$$\bar{z}_R W_R = \tilde{z}_1 W_1 + \tilde{z}_2 W_2 + \tilde{z}_3 W_3 + \quad \tilde{z}_n W_n$$

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i W_i}{\sum_{i=1}^{n} W_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i W_i}{\sum_{i=1}^{n} W_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i W_i}{\sum_{i=1}^{n} W_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of gravit

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i weight of the i^{th} particle

Center of Mass

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i m_i}{\sum_{i=1}^{n} m_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i m_i}{\sum_{i=1}^{n} m_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i m_i}{\sum_{i=1}^{n} m_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of mas

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i mass of the i^{th} particle

Center of Gravity and Centroid for a Body

Consider a body to be a system of an infinite number of particles

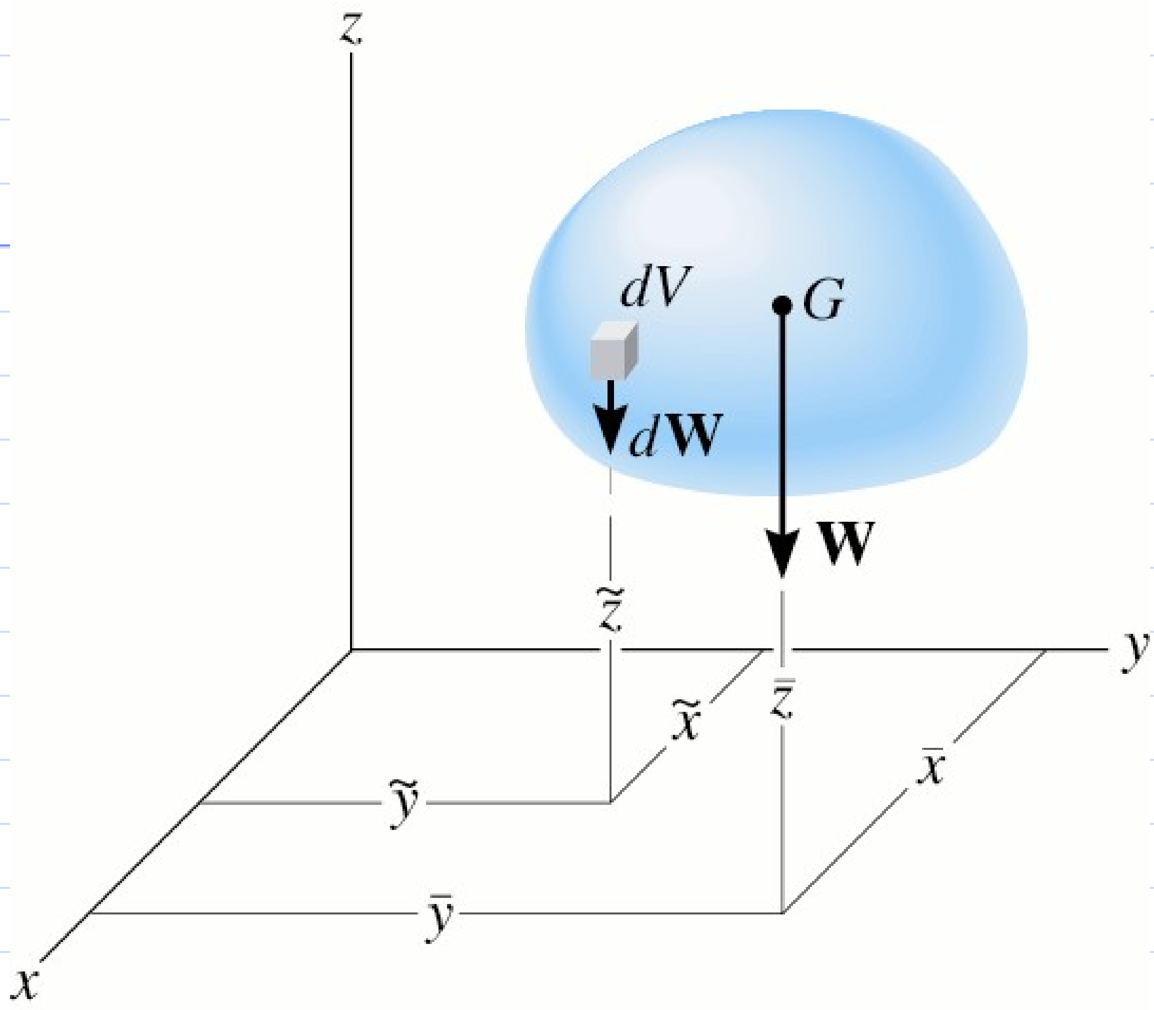


Figure 09.02

$$\bar{x} = \frac{\sum_{i=1}^{\infty} \tilde{x}_i W_i}{\sum_{i=1}^{\infty} W_i} \qquad \bar{y} = \frac{\sum_{i=1}^{\infty} \tilde{y}_i W_i}{\sum_{i=1}^{\infty} W_i} \qquad \bar{z} = \frac{\sum_{i=1}^{\infty} \tilde{z}_i W_i}{\sum_{i=1}^{\infty} W_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of gravit

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i weight of the i^{th} particle

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

γ - specific weight of the body

(*The weight per unit volume*)

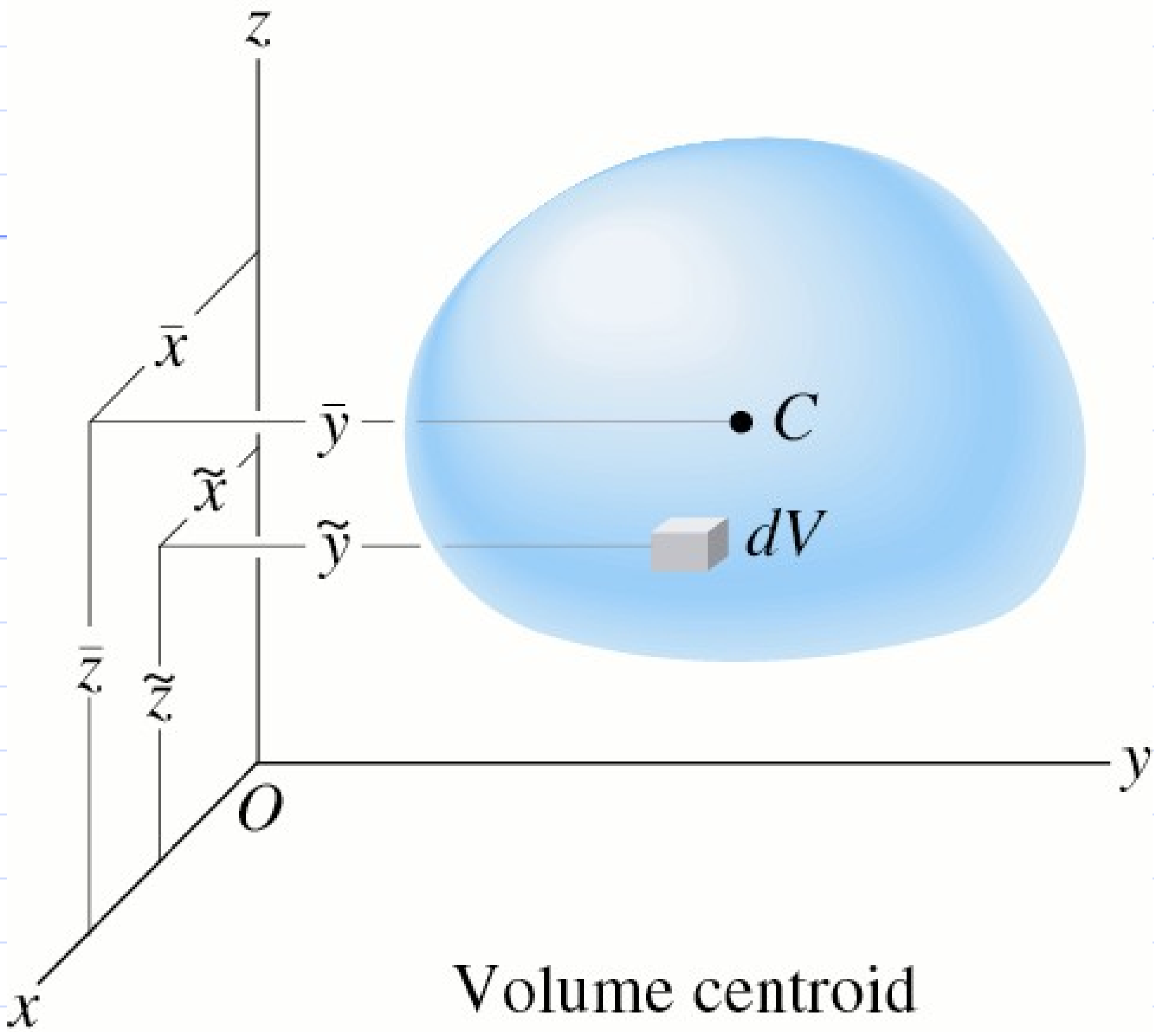
$$dW = \gamma\, dV$$

Center of Gravity of a Body

$$\bar{x} = \frac{\int_V \tilde{x}\gamma dV}{\int_V \gamma dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\gamma dV}{\int_V \gamma dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\gamma dV}{\int_V \gamma dV}$$

CENTROID

The centroid C is a point which defines the geometric center of an object. Its location can be determined by formulas similar to those used for center of gravity or center of mass.

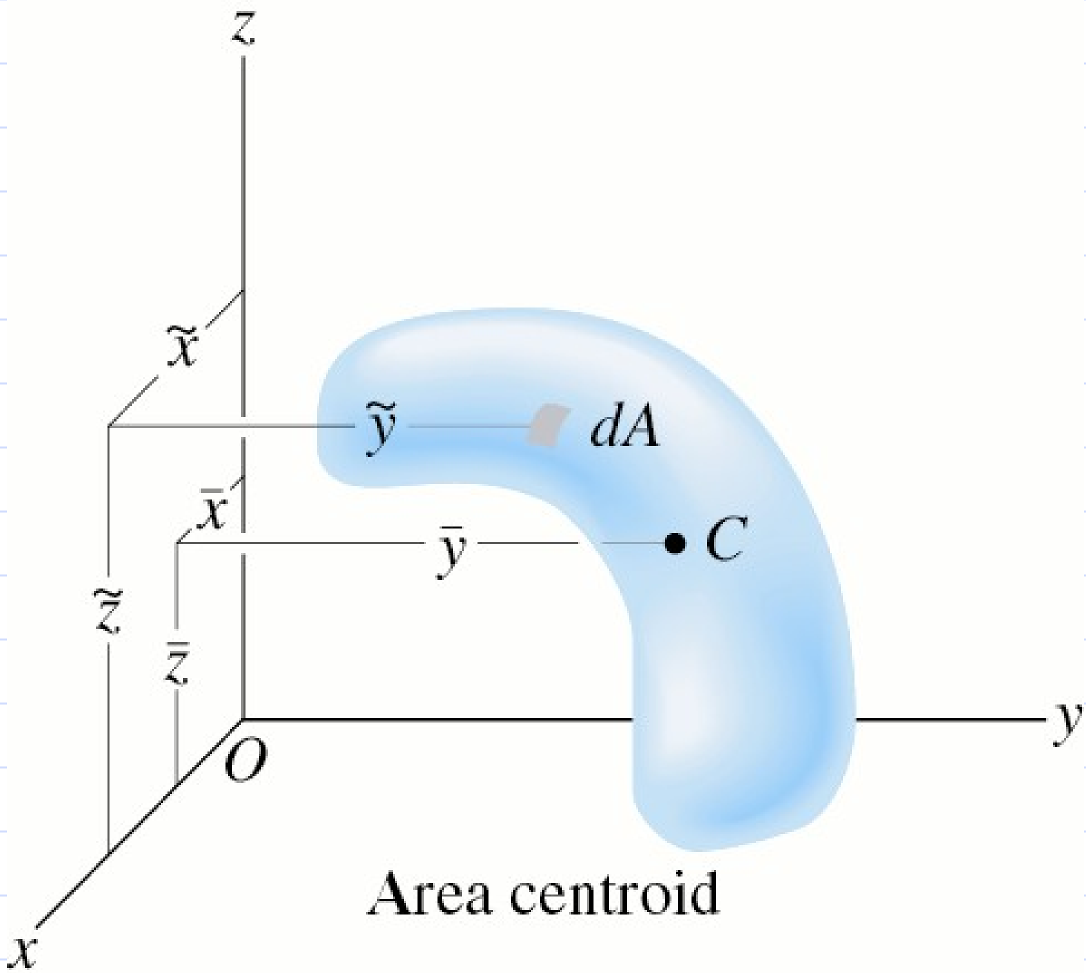


Volume centroid

Figure 09.03

Centroid of a Volume

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$



Area centroid

Figure 09.04

Centroid of an Area

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} \qquad \bar{z} = \frac{\int_A \tilde{z}\, dA}{\int_A dA}$$

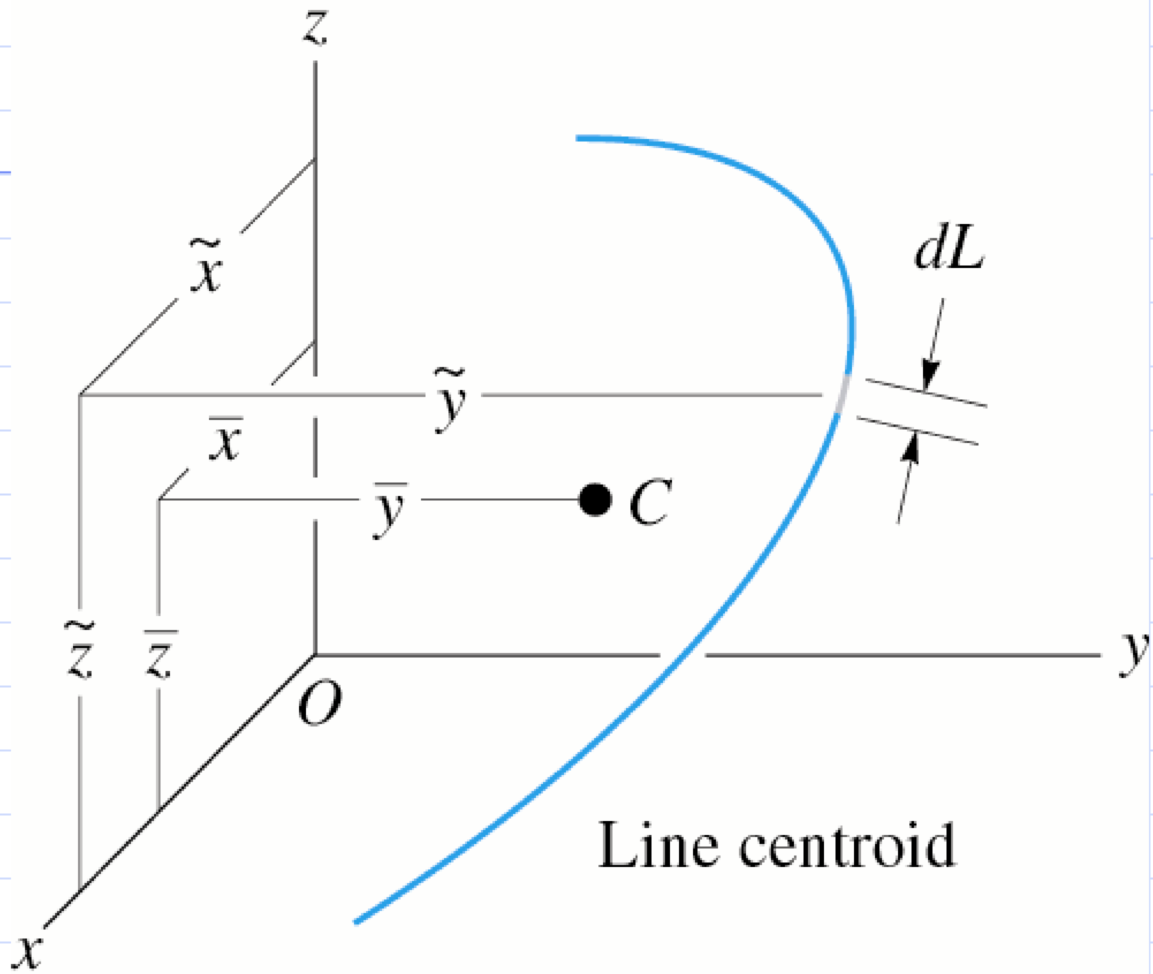


Figure 09.05

Centroid of a Line

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} \qquad \bar{z} = \frac{\int_L \tilde{z}\, dL}{\int_L dL}$$

Figure 09.05.01(C)

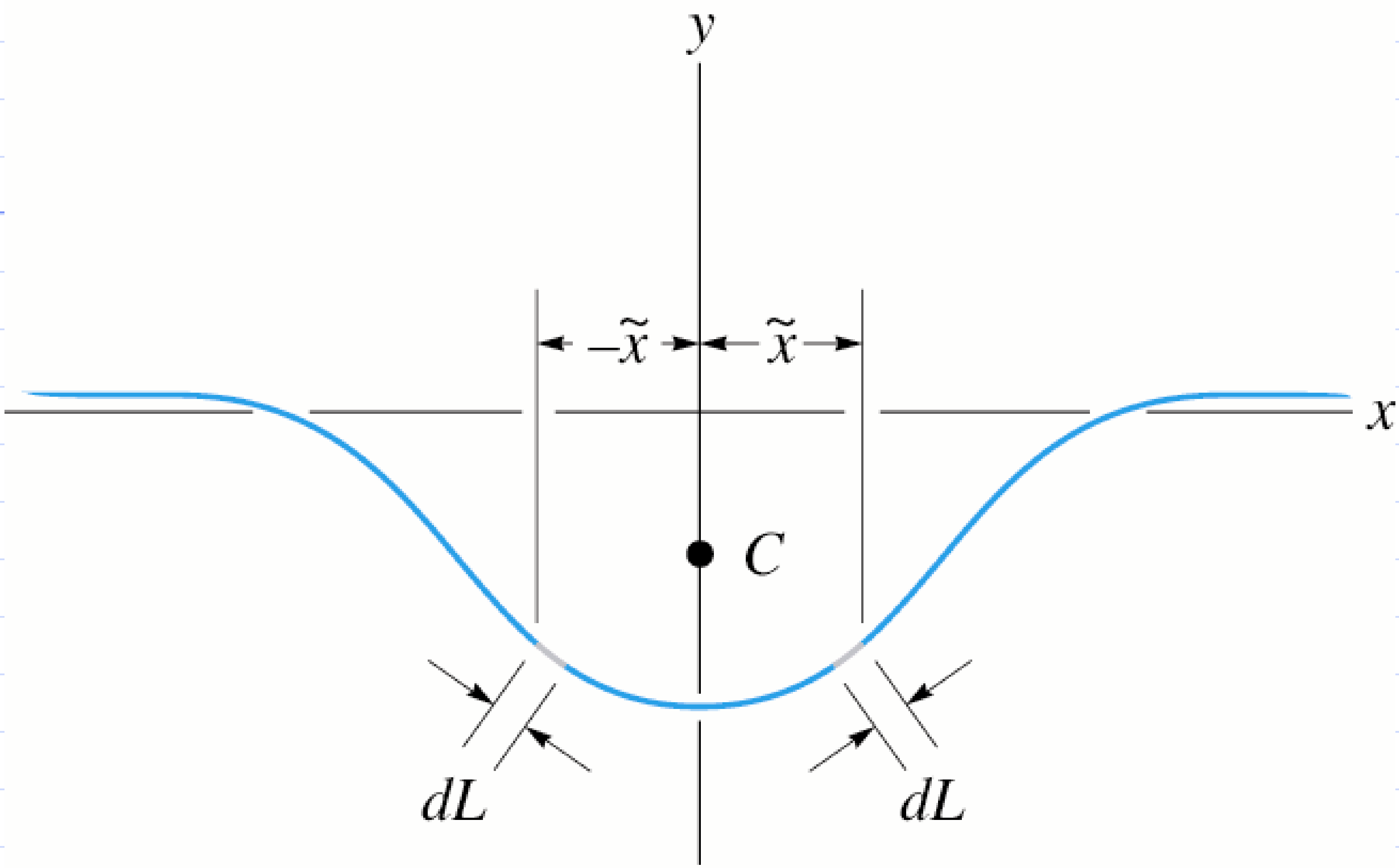


Figure 09.06

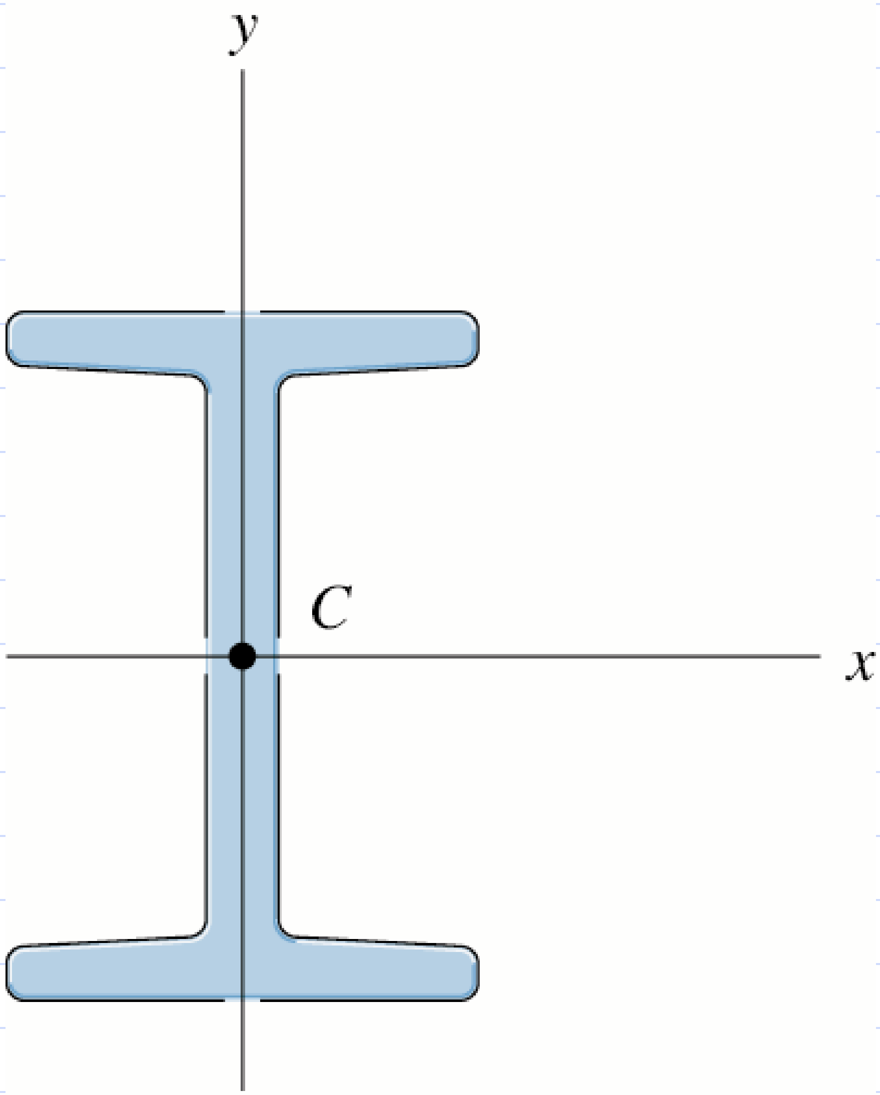


Figure 09.07

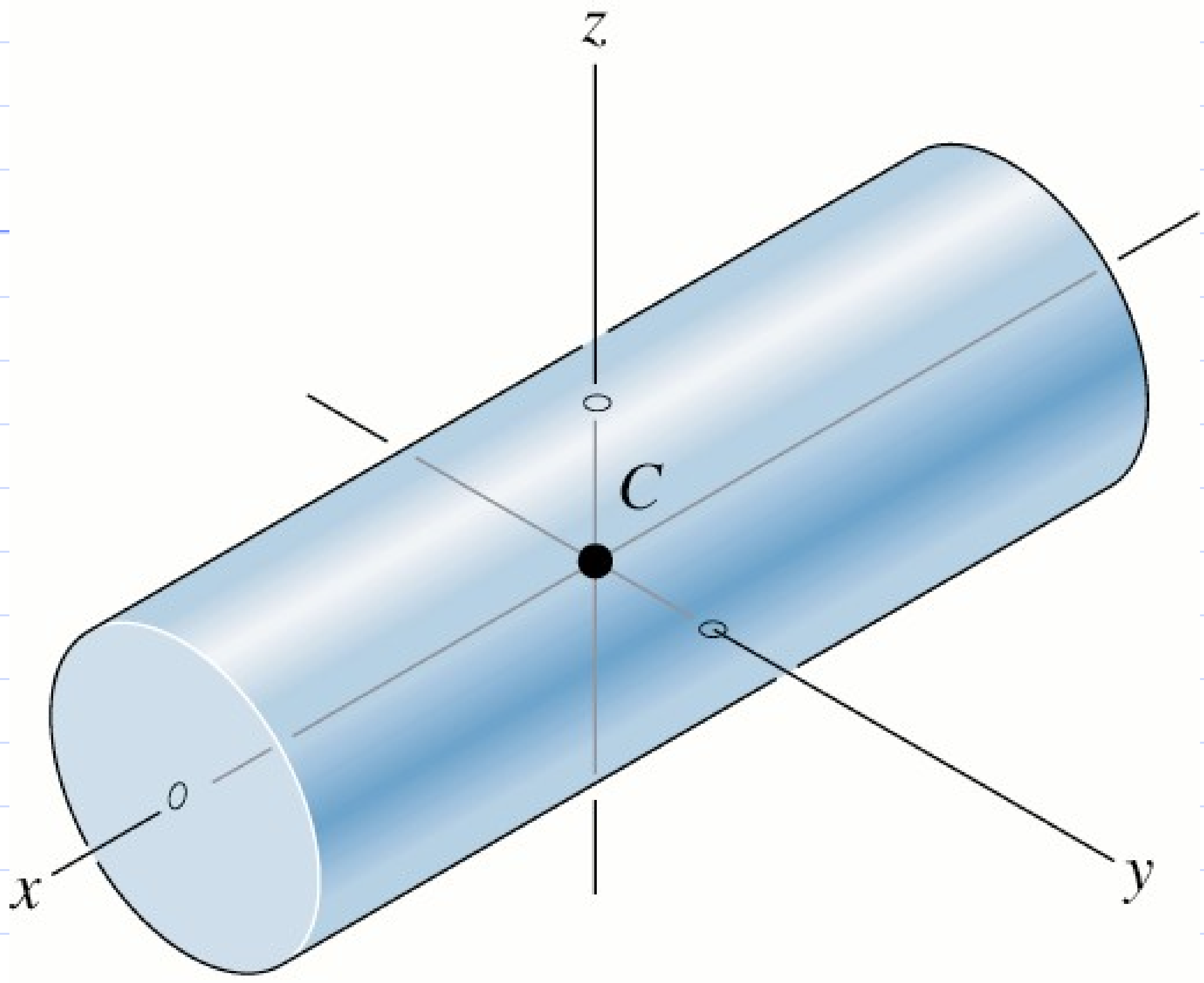


Figure 09.08

PROBLEM

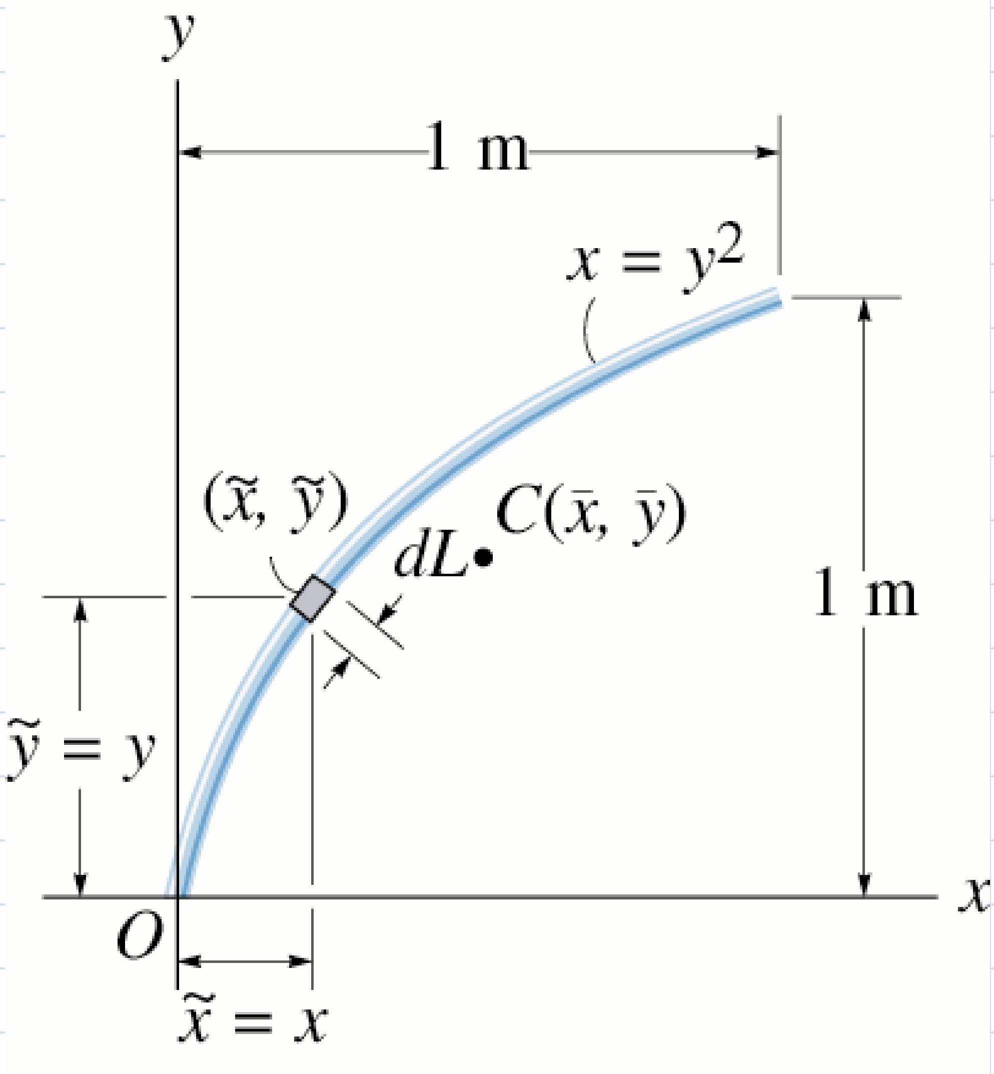


Figure 09.09

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$dL = \left(\sqrt{\left(\frac{dx}{dy} \right)^2 + 1} \right) dy$$

$$x = y^2$$

$$\frac{dx}{dy} = 2y$$

$$dL = \left(\sqrt{(2y)^2 + 1} \right) dy$$

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{\int_0^1 x\left(\sqrt{(2y)^2 + 1}\right) dy}{\int_0^1 \left(\sqrt{(2y)^2 + 1}\right) dy}$$

$$x = y^2$$

$$\bar{x} = \frac{\int_0^1 y^2\left(\sqrt{4y^2 + 1}\right) dy}{\int_0^1 \left(\sqrt{4y^2 + 1}\right) dy} = \frac{0.6063}{1.479} = 0.410\ \text{m}$$

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{\int_0^1 y\left(\sqrt{(2y)^2 + 1}\right) dy}{\int_0^1 \left(\sqrt{(2y)^2 + 1}\right) dy}$$

$$\bar{y} = \frac{\int_0^1 y\left(\sqrt{4y^2 + 1}\right) dy}{\int_0^1 \left(\sqrt{4y^2 + 1}\right) dy} = \frac{0.8484}{1.479} = 0.574\,\mathrm{m}$$

PROBLEM

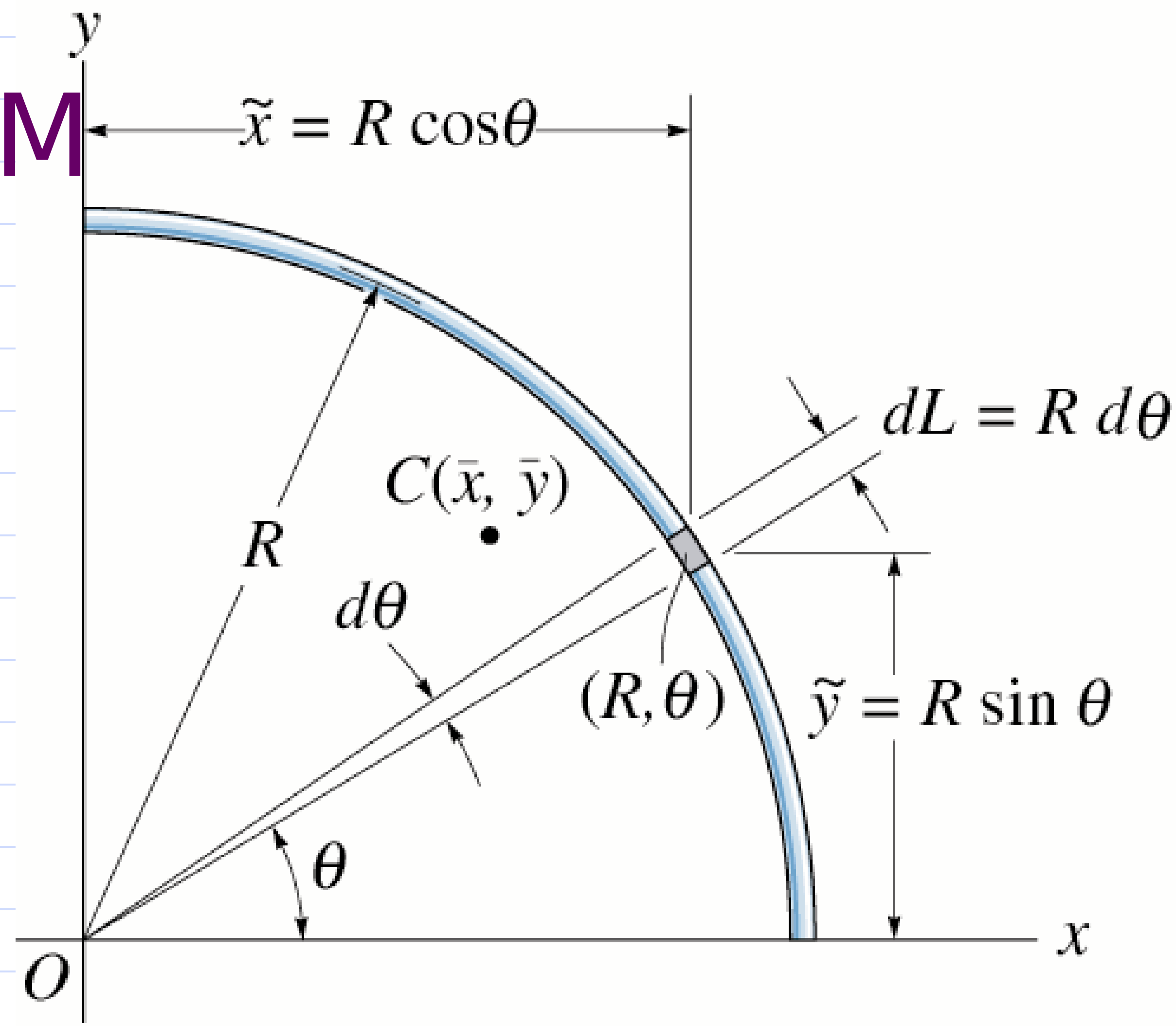
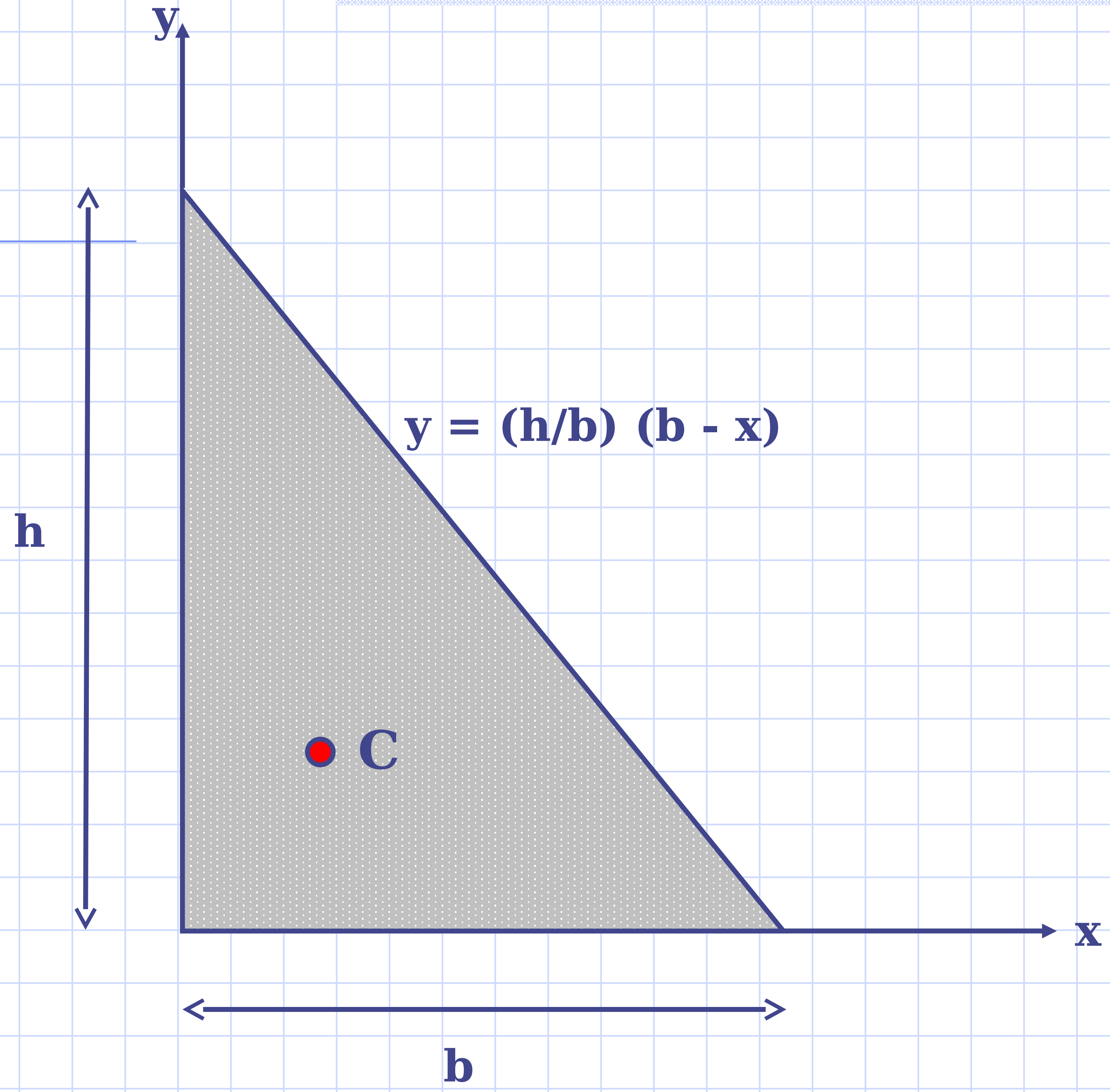


Figure 09.10

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{\frac{\pi}{2}} (R\cos\theta)\,R\,d\theta}{\int_0^{\frac{\pi}{2}} R\,d\theta} = \frac{R^2 \int_0^{\frac{\pi}{2}} \cos\theta\,d\theta}{R \int_0^{\frac{\pi}{2}} d\theta} = \frac{2R}{\pi}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{\frac{\pi}{2}} (R\sin\theta)\,R\,d\theta}{\int_0^{\frac{\pi}{2}} R\,d\theta} = \frac{R^2 \int_0^{\frac{\pi}{2}} \sin\theta\,d\theta}{R \int_0^{\frac{\pi}{2}} d\theta} = \frac{2R}{\pi}$$



y
y = (h/b) (b - x)
h
C
x
b

Strip Method

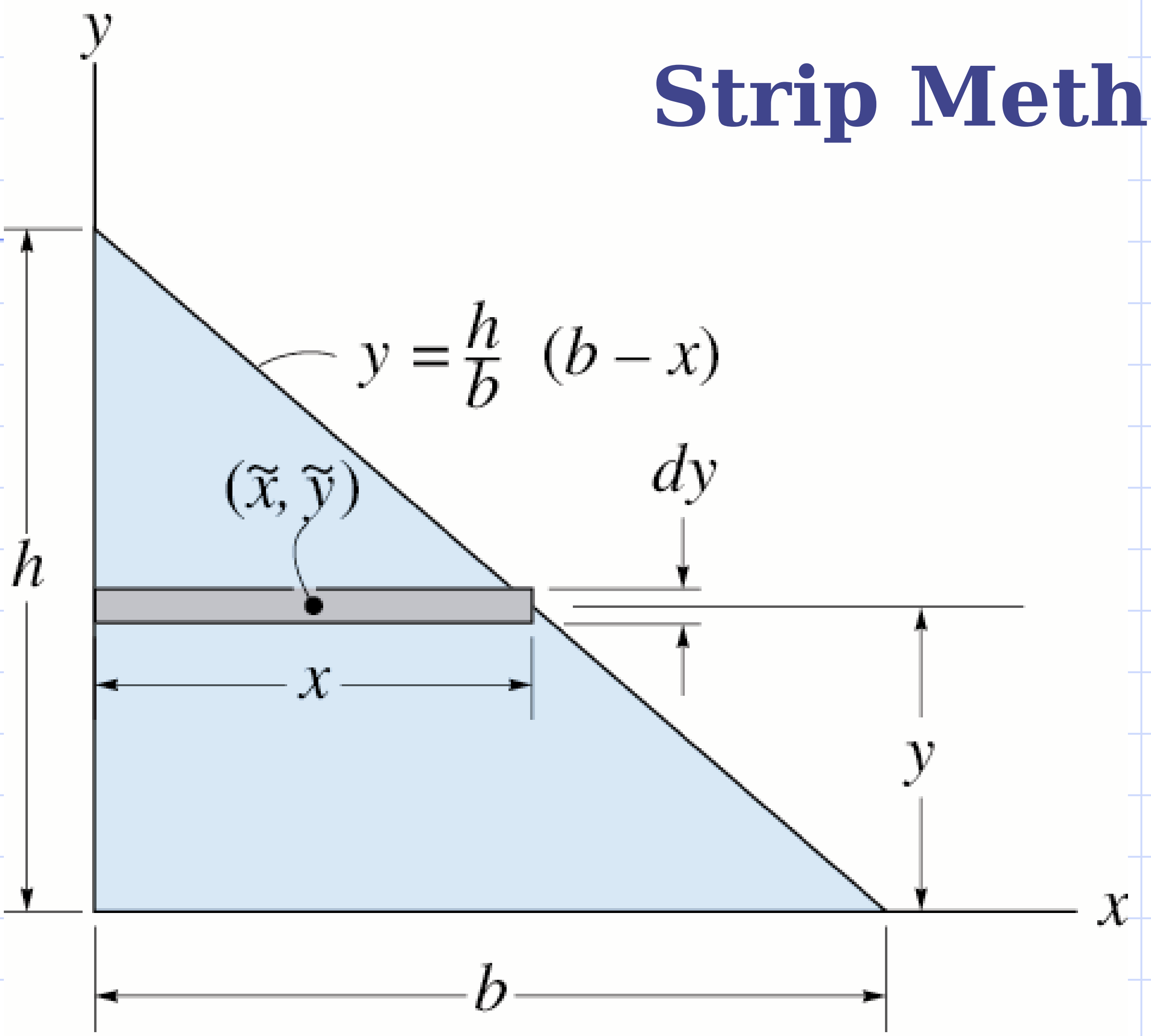


Figure 09.11

$$dA = x\,dy$$

$$dA = \frac{b}{h}(h - y)\,dy$$

$$\tilde{x} = \frac{1}{2}\left(\frac{b}{h}(h - y)\right)$$

$$\tilde{y} = y$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(\frac{b}{h}(h-y)\right)dy}{\int_0^h \left(\frac{b}{h}(h-y)\right)dy}$$

$$\bar{y} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{h}{3}$$

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^h \frac{1}{2}\left(\frac{b}{h}(h-y)\right)\left(\frac{b}{h}(h-y)\right) dy}{\int_0^h \left(\frac{b}{h}(h-y)\right) dy}$$

$$\bar{x} = \frac{\frac{1}{6}\, b^2\, h}{\frac{1}{2}\, b\, h} = \frac{b}{3}$$

PROBLEM

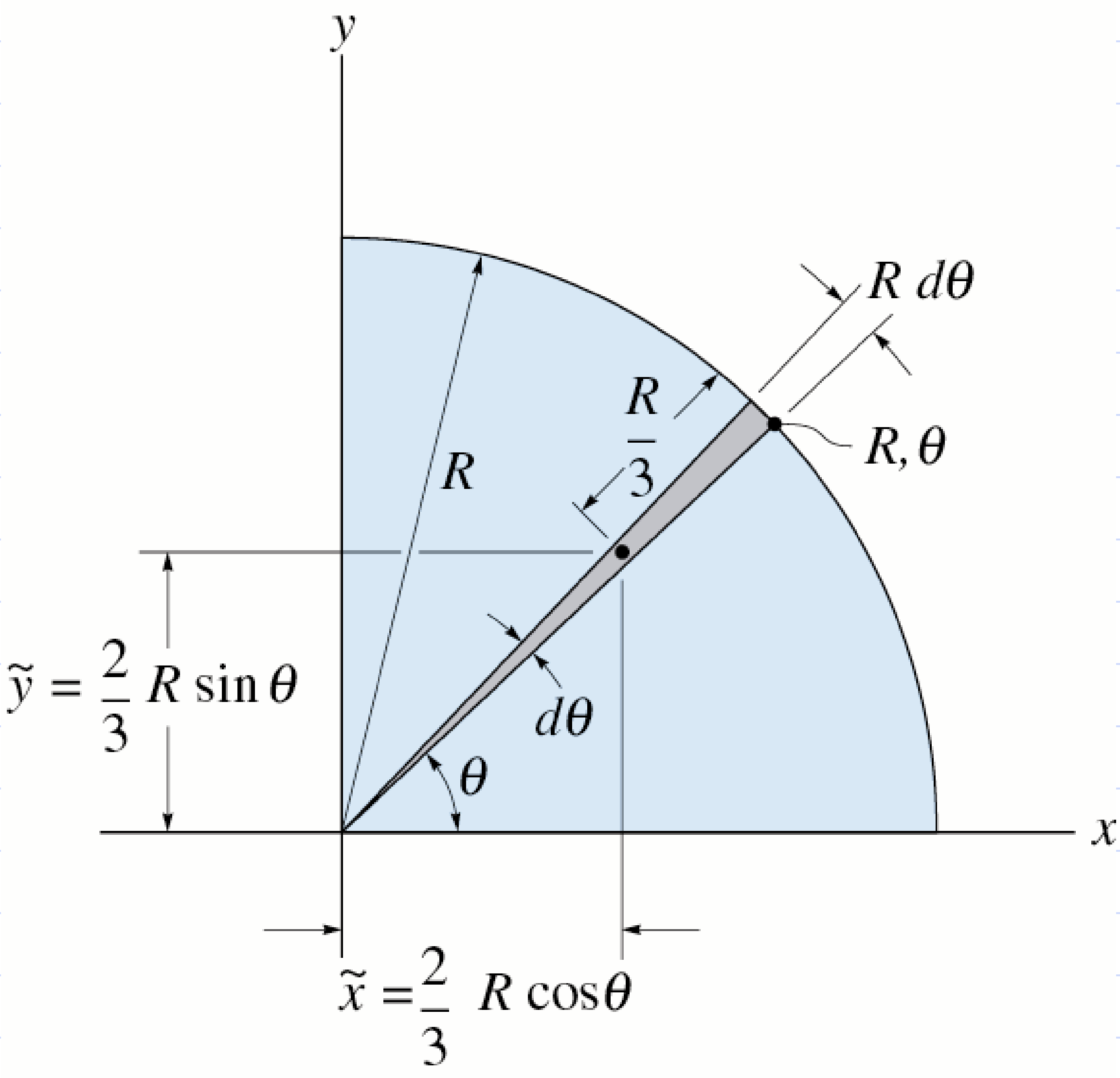


Figure 09.12(a)

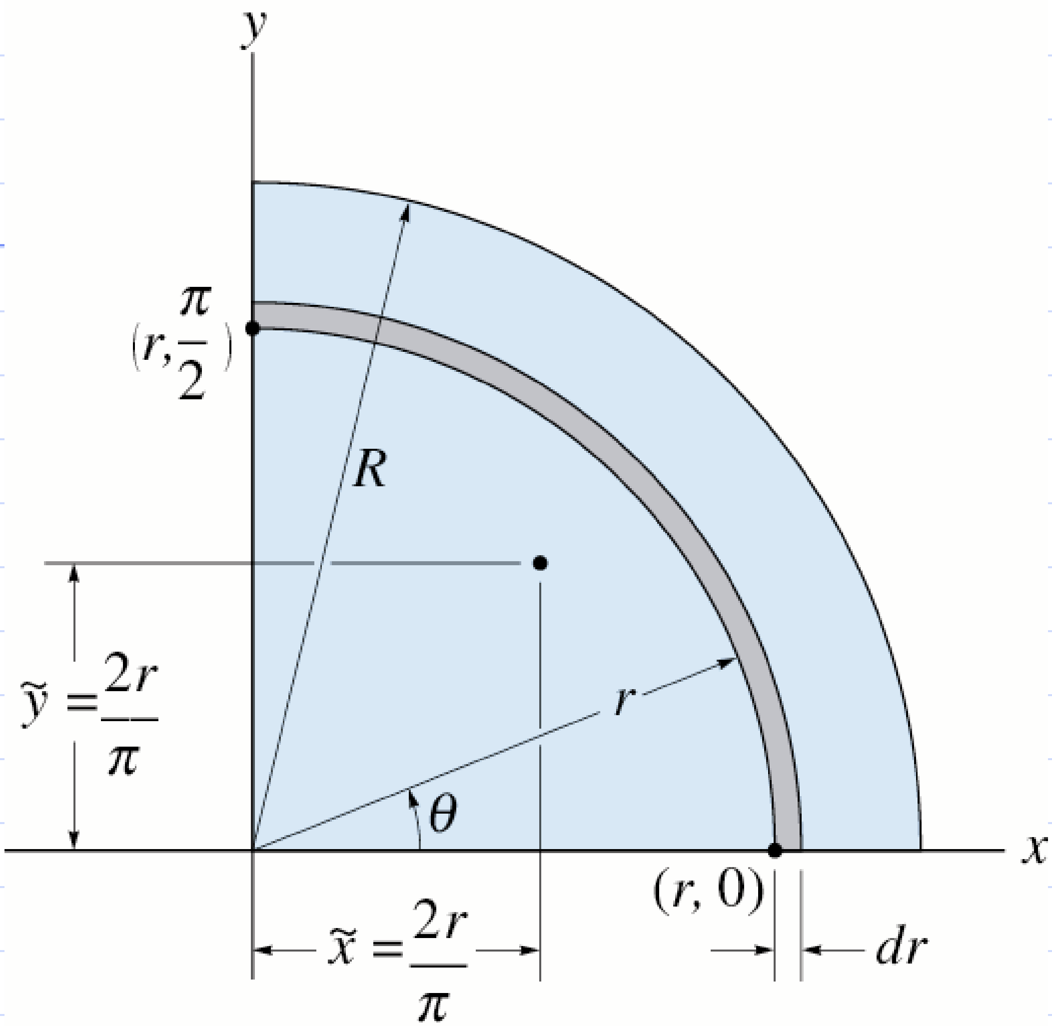


Figure 09.12(b)

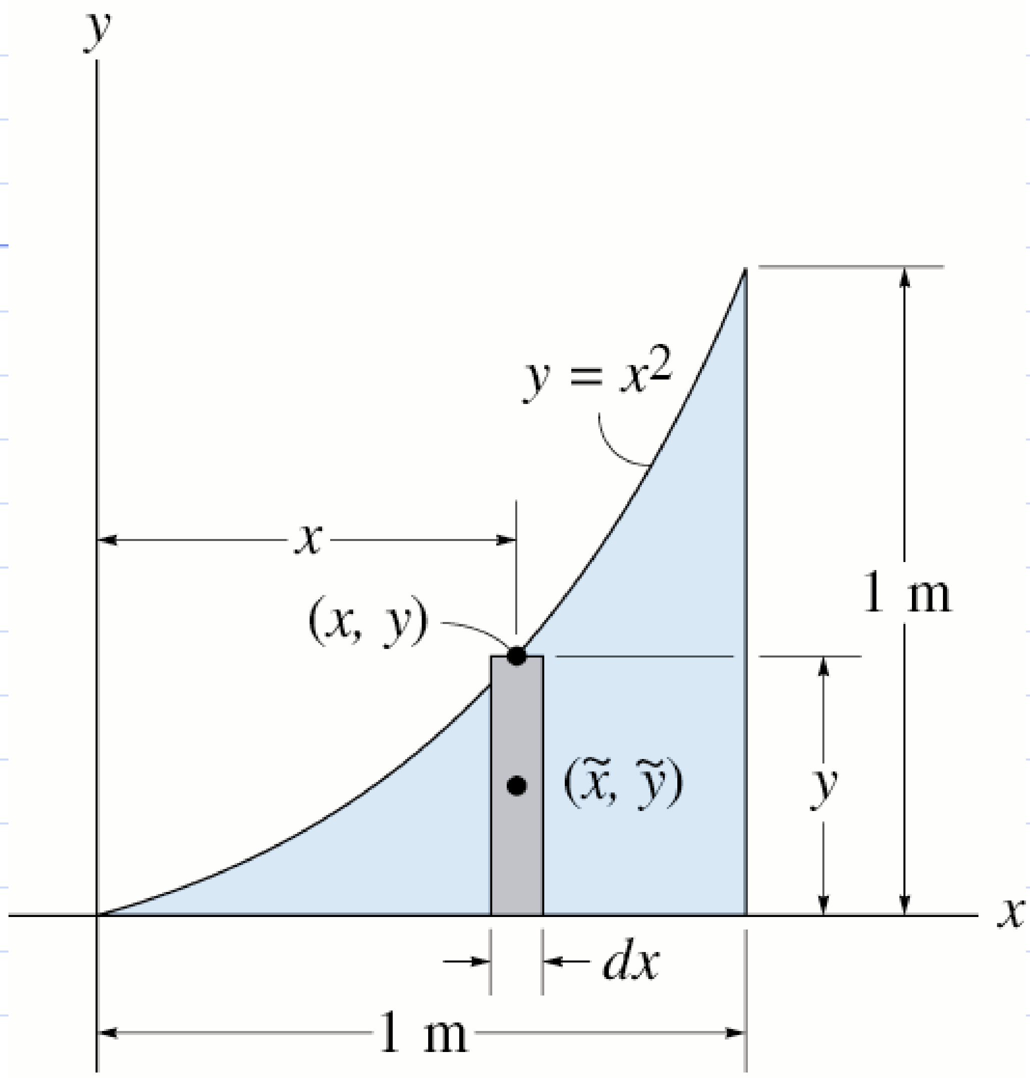


Figure 09.13(a)

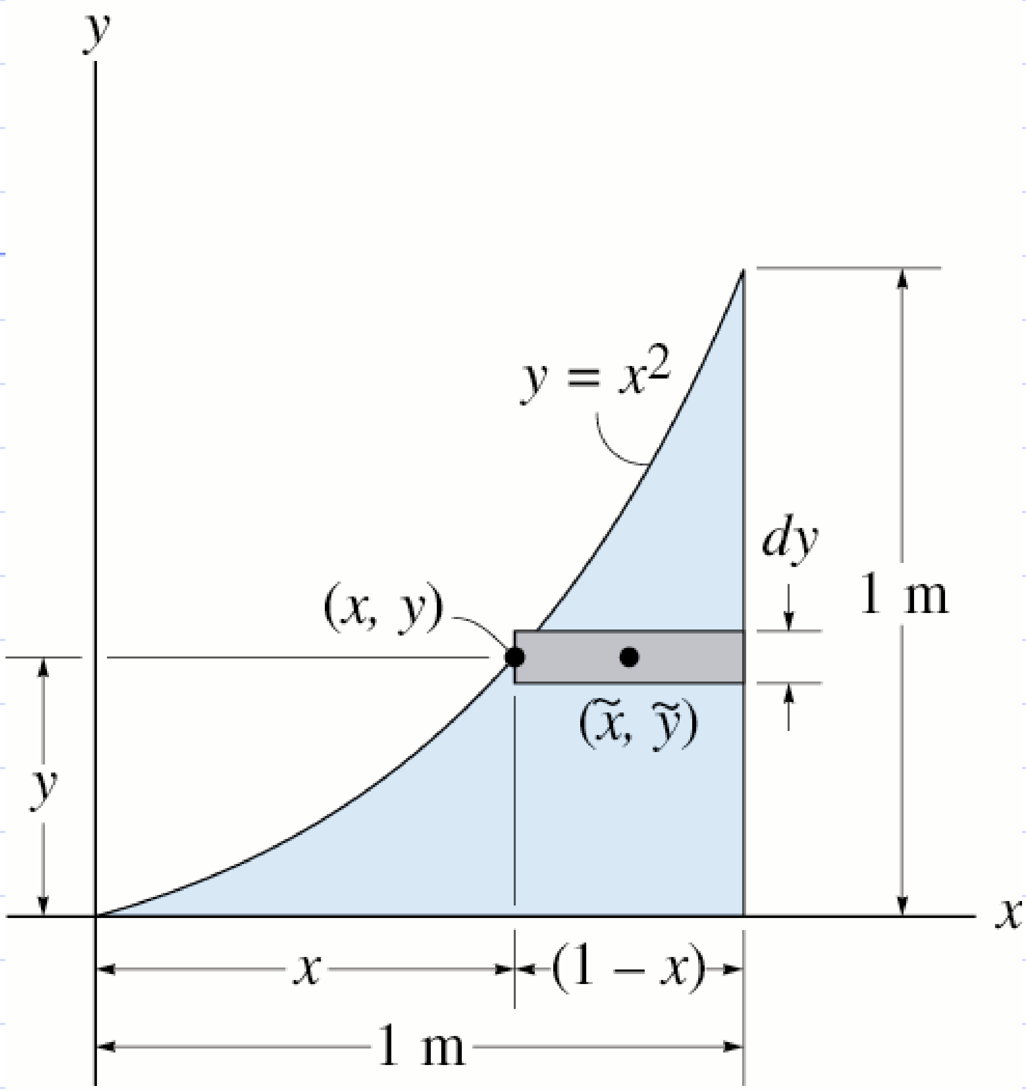


Figure 09.13(b)

Composite Bodies

Figure 09.16.01(C)

Composite Bodies

If a body is made up of several simpler bodies then a special technique can be used.

Procedure

- Divide body into several subparts.
- If the body has a hole or cutout, treat that as negative area.
- Centroid will lie on line of symmetry.
- Create Table and calculate centroid.

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i A_i}{\sum_{i=1}^{n} A_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of gravit

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i weight of the i^{th} particle

Body	Area	x_c	y_c	$x_c A$	$y_c A$

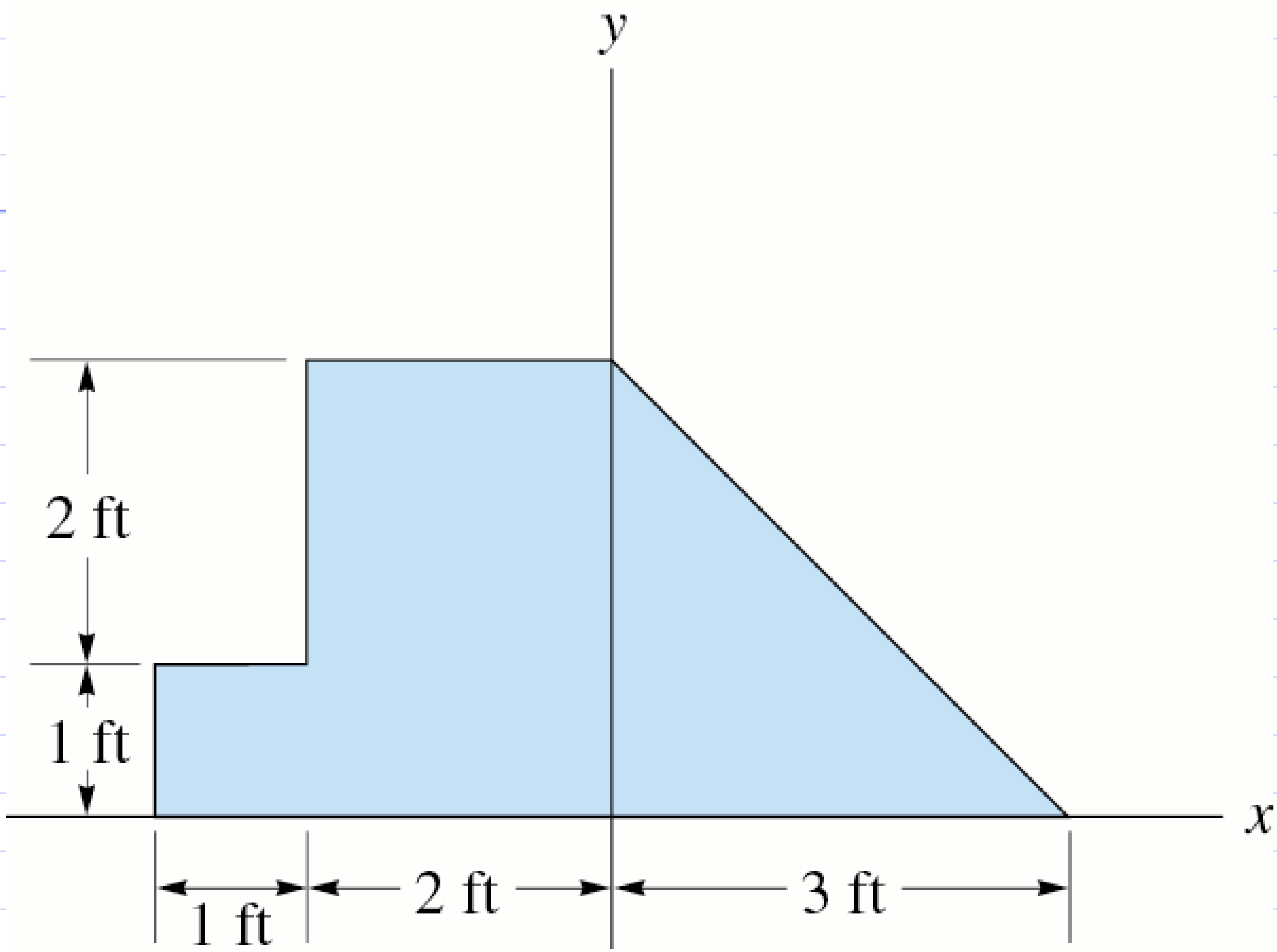


Figure 09.18(a)

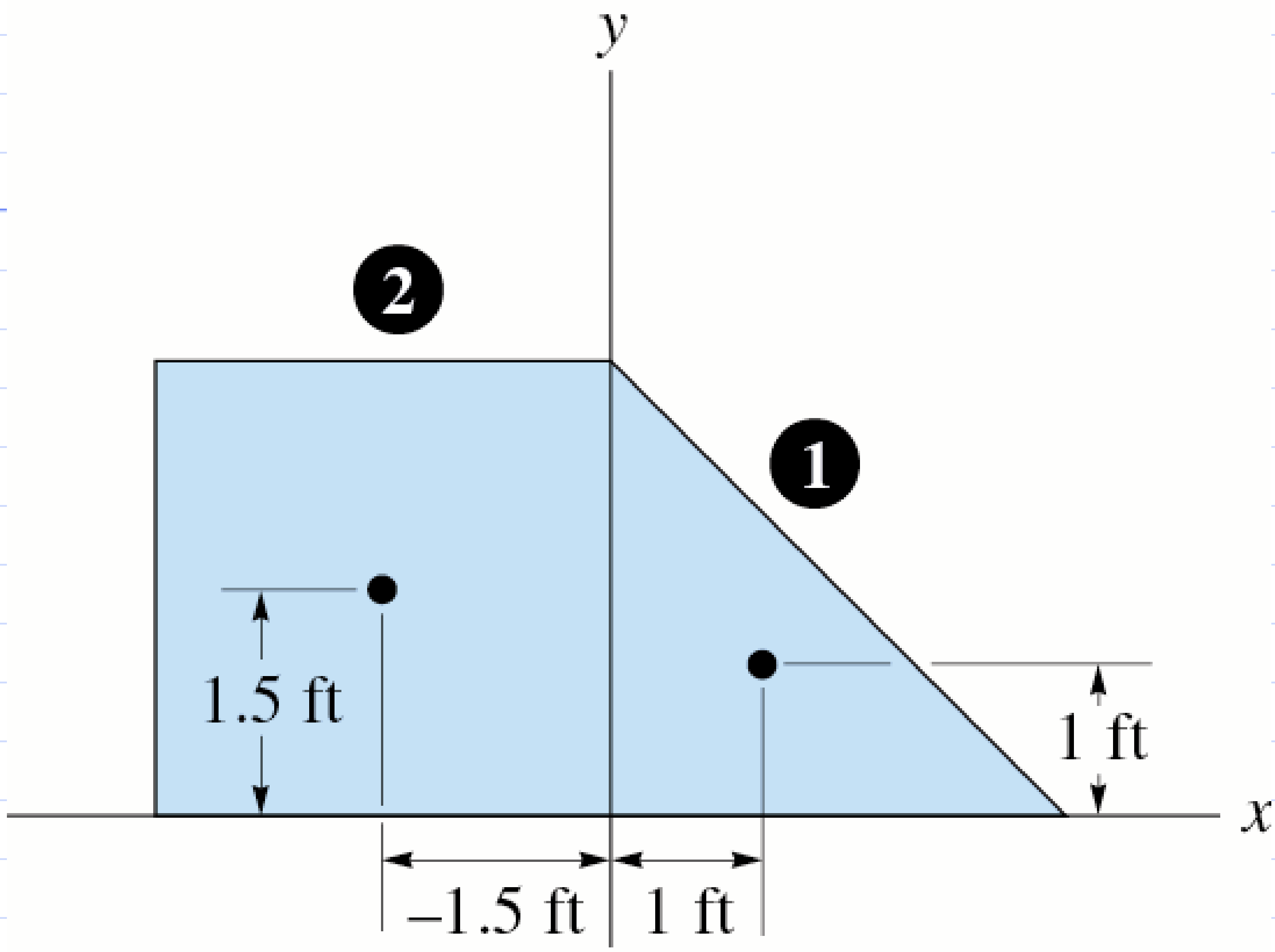


Figure 09.18(b1)

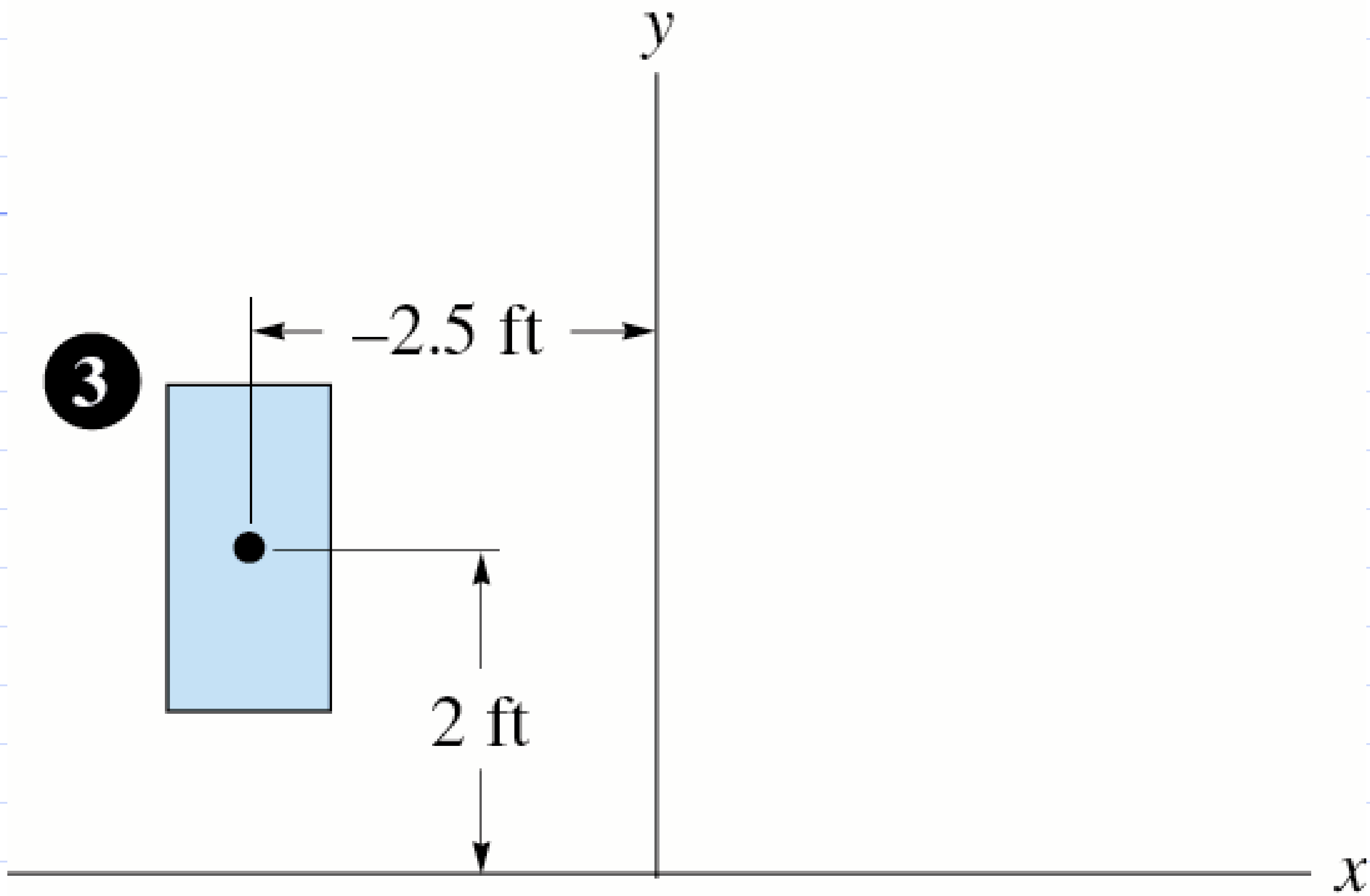
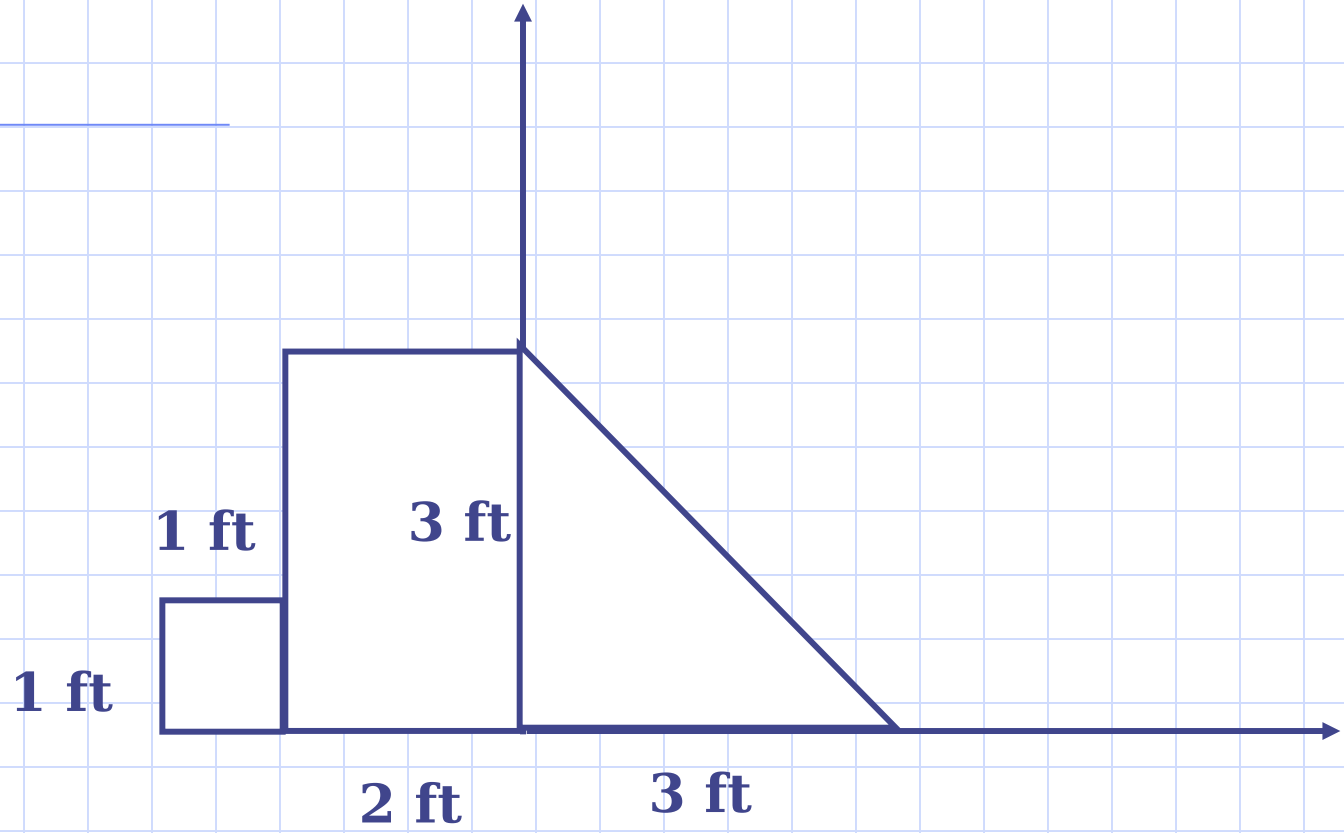
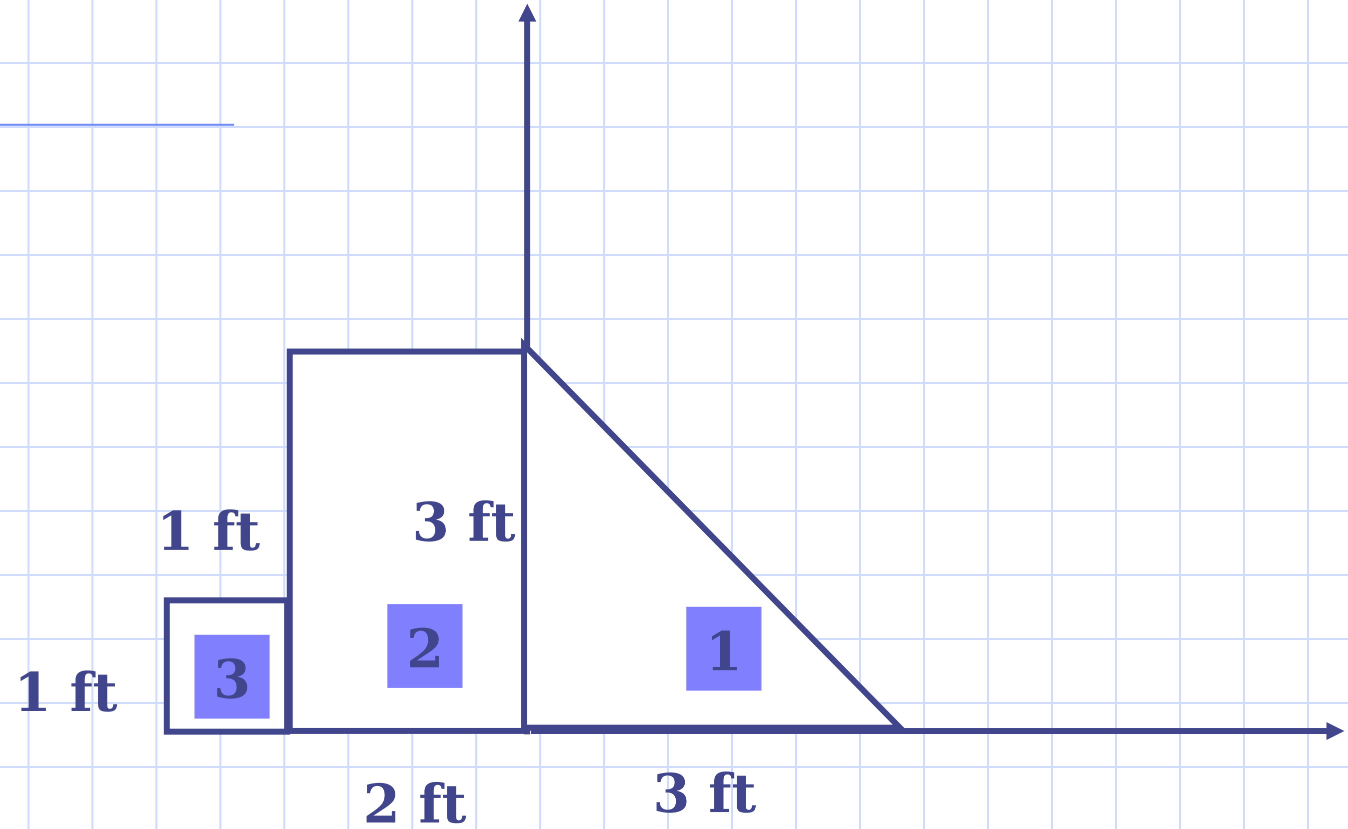


Figure 09.18(b2)



Locate Centroid of the Composite Area

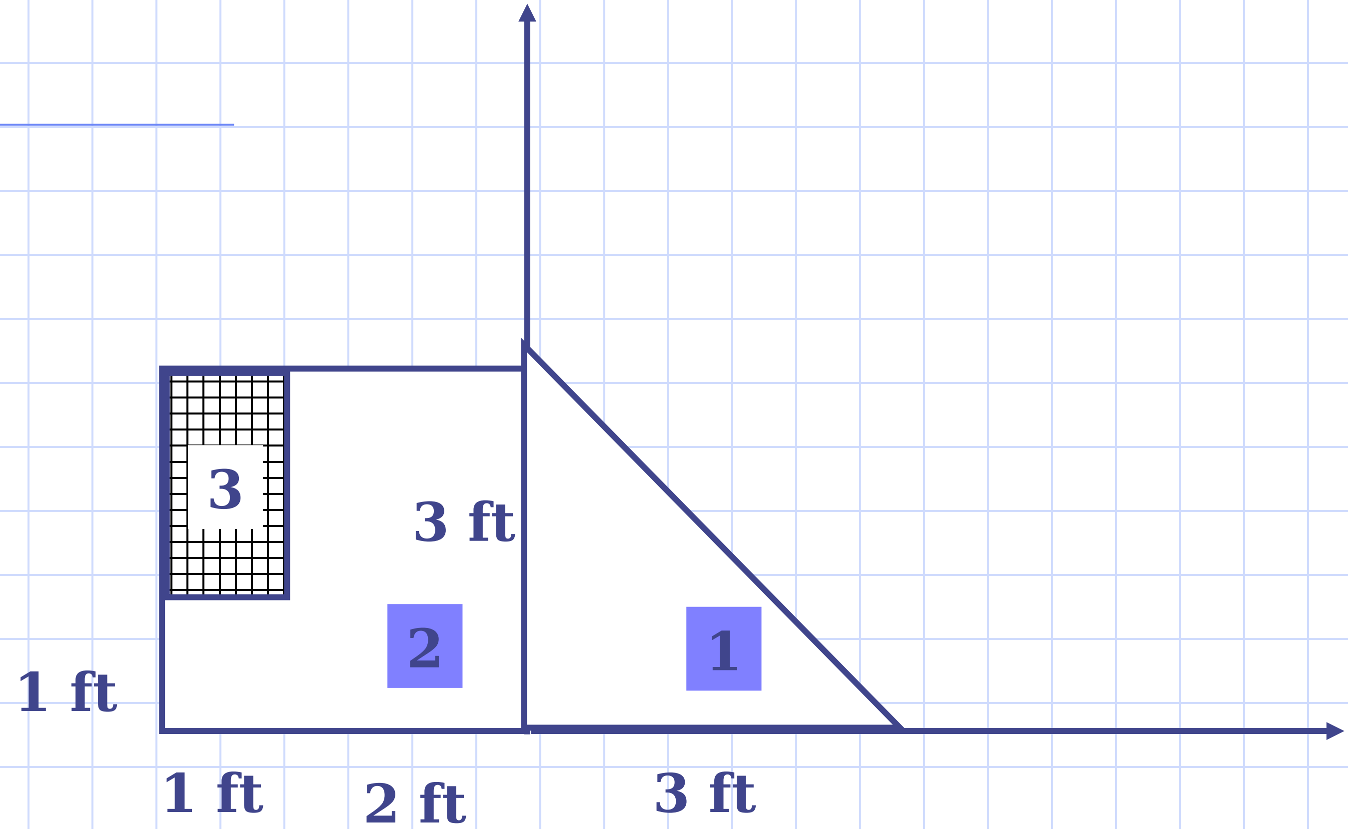

1 ft
3 ft
2
1
3
1 ft
2 ft
3 ft

Segment	A (ft^2)	x	y	xA	yA
1	4.5	1	1	4.5	4.5
2	6	-1	1.5	-6	9
3	1	-2.5	0.5	-2.5	0.5

$\Sigma A = 11.5$ $\Sigma xA = -4$ $\Sigma xA = 14$

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{-4}{11.5} = -0.348 \text{ ft}$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{14}{11.5} = 1.22 \text{ ft}$$



3
3 ft
2
1
1 ft
1 ft
2 ft
3 ft

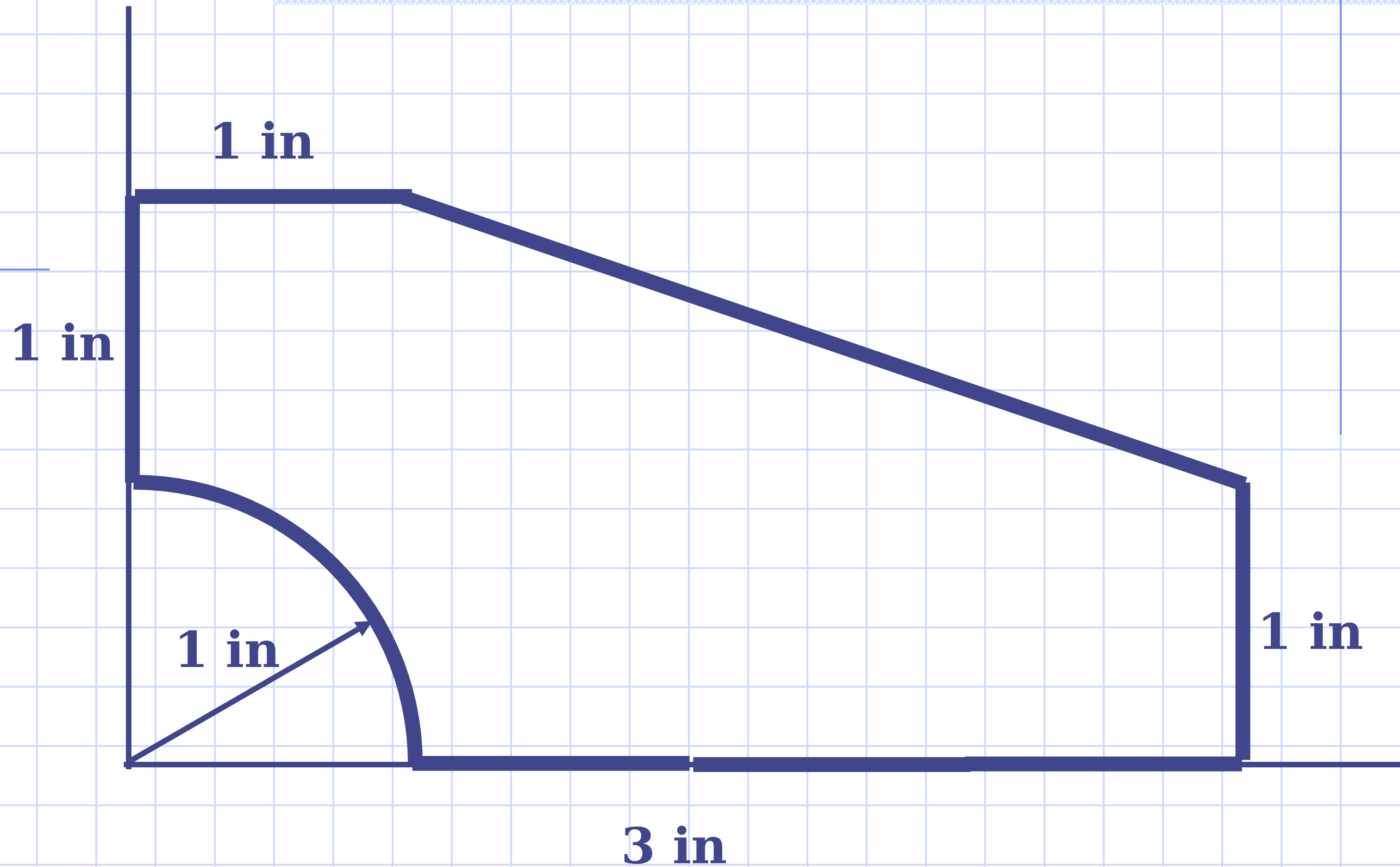
Segement	A (ft²)	x	y	xA	yA
1	4.5	1	1	4.5	4.5
2	9	-1.5	1.5	-13.5	13.5
3	-2.5	-2.5	2	5	-4

Σ A = 11.5 Σ xA = -4 Σ xA = 14

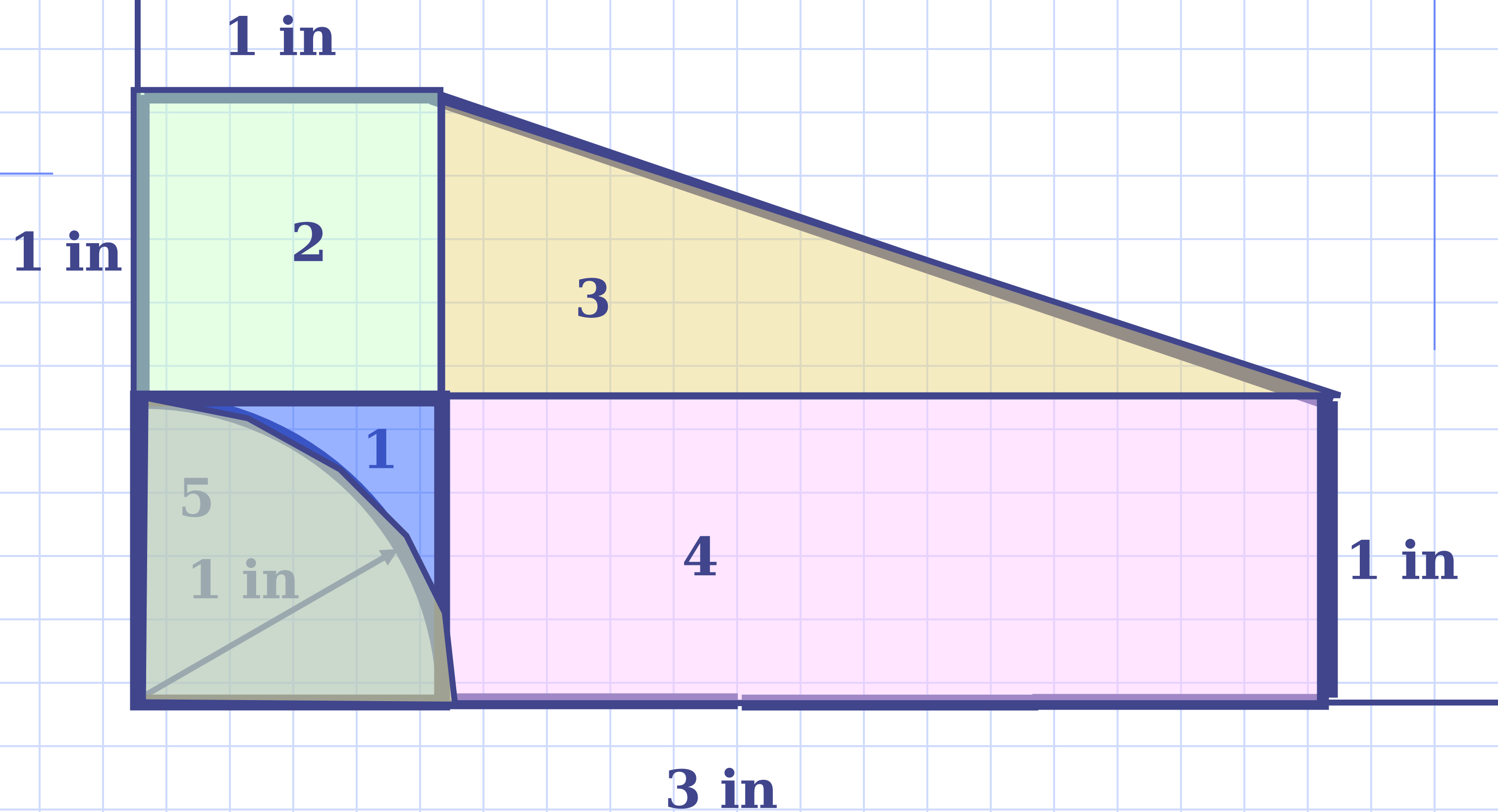
$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{-4}{11.5} = -0.348\ \text{ft}$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{14}{11.5} = 1.22\ \text{ft}$$

9.55



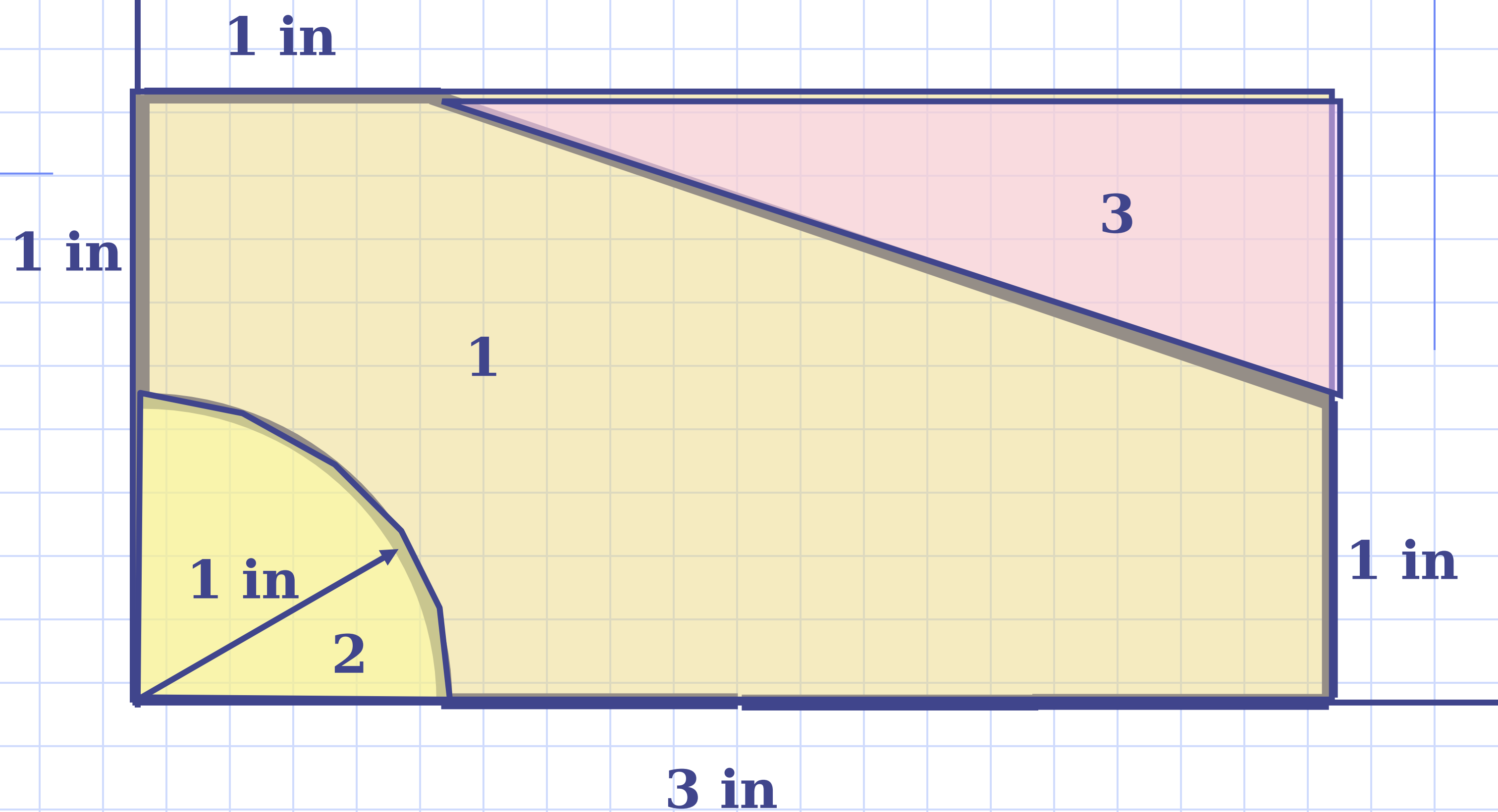
9.55



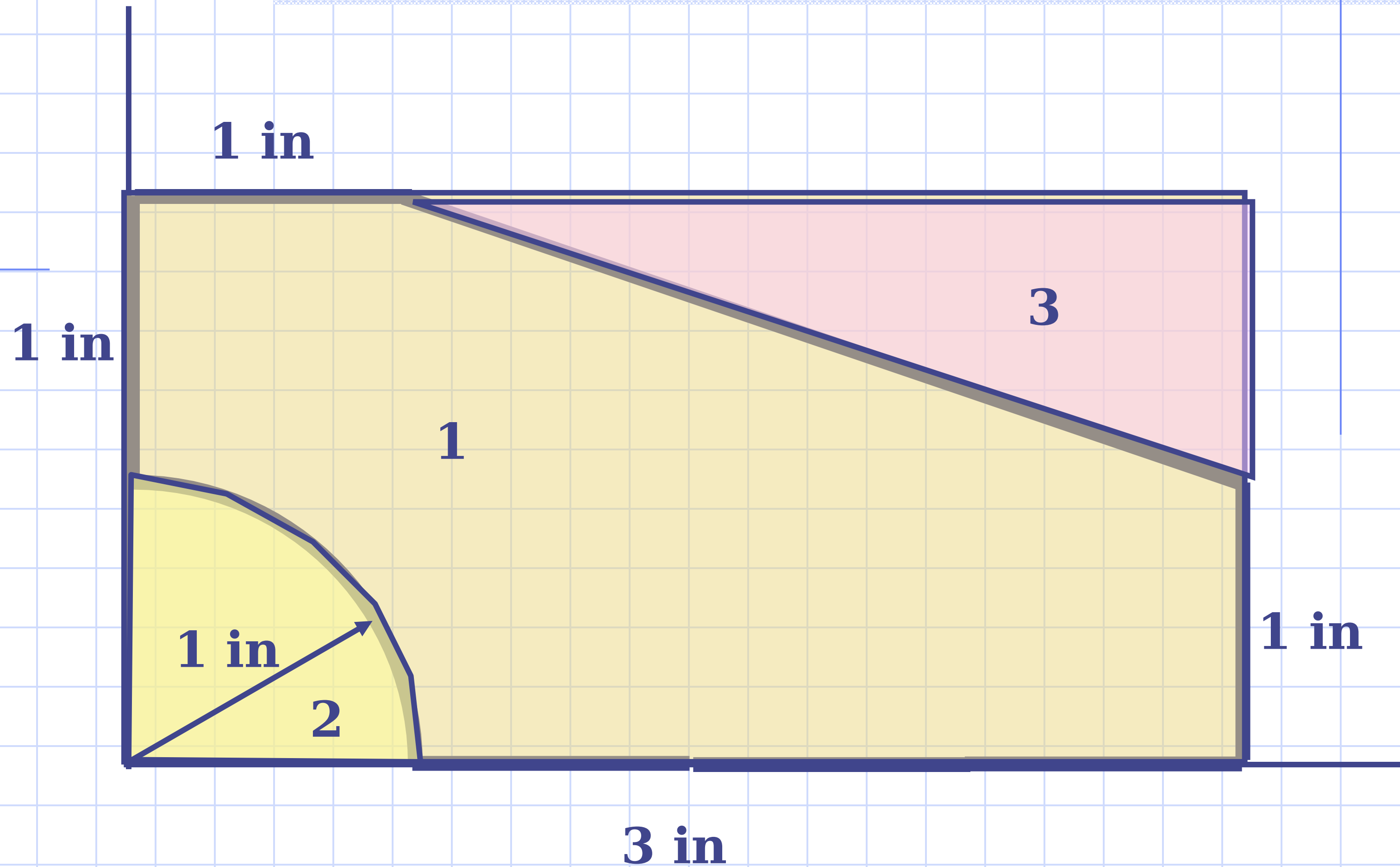
Break into sub-areas

Segment	Area	x	y	xA	yA
1.00000	1.00000	0.50000	0.50000	0.50000	0.50000
2.00000	1.00000	0.50000	1.50000	0.50000	1.50000
3.00000	1.50000	2.00000	1.33333	3.00000	2.00000
4.00000	3.00000	2.50000	0.50000	7.50000	1.50000
5.00000	-0.78540	0.42441	0.42441	-0.33333	-0.33333
	5.71460			11.16667	5.16667
	x=	1.95406			
	y=	0.90412			

9.55



9.55



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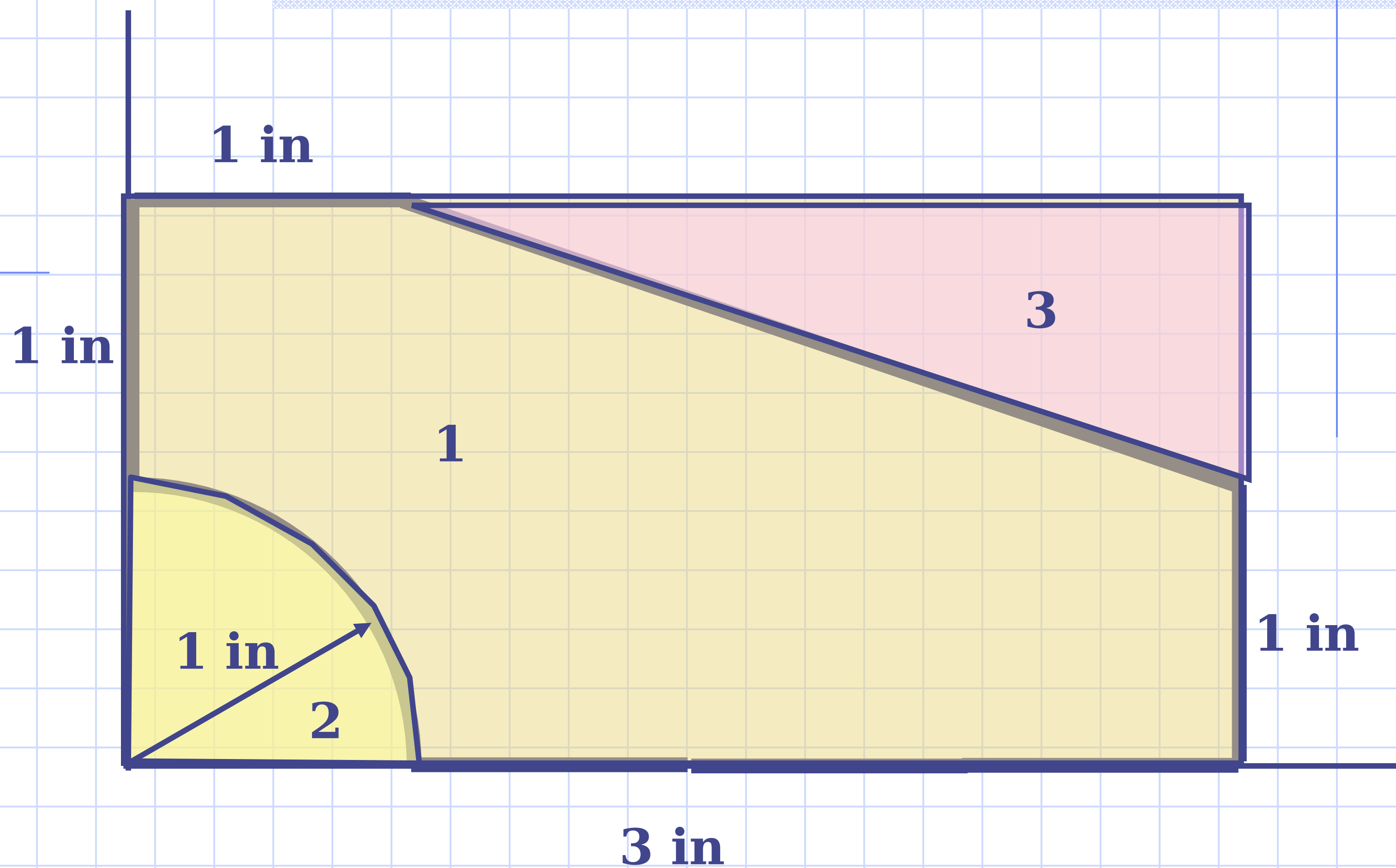
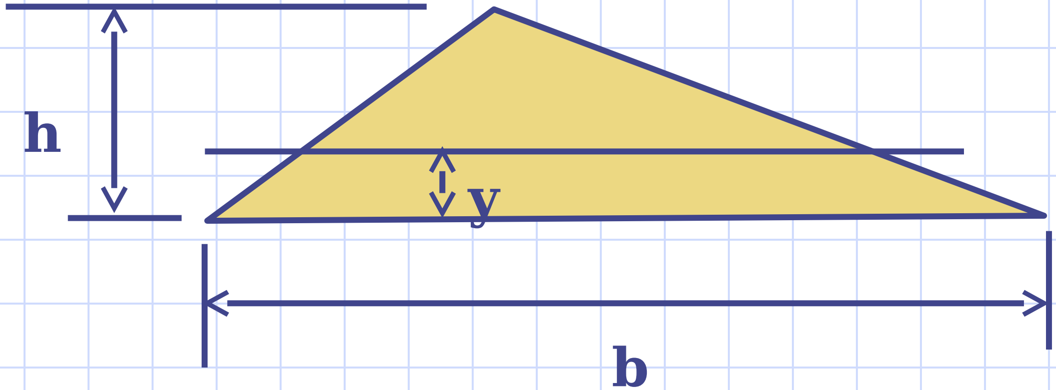
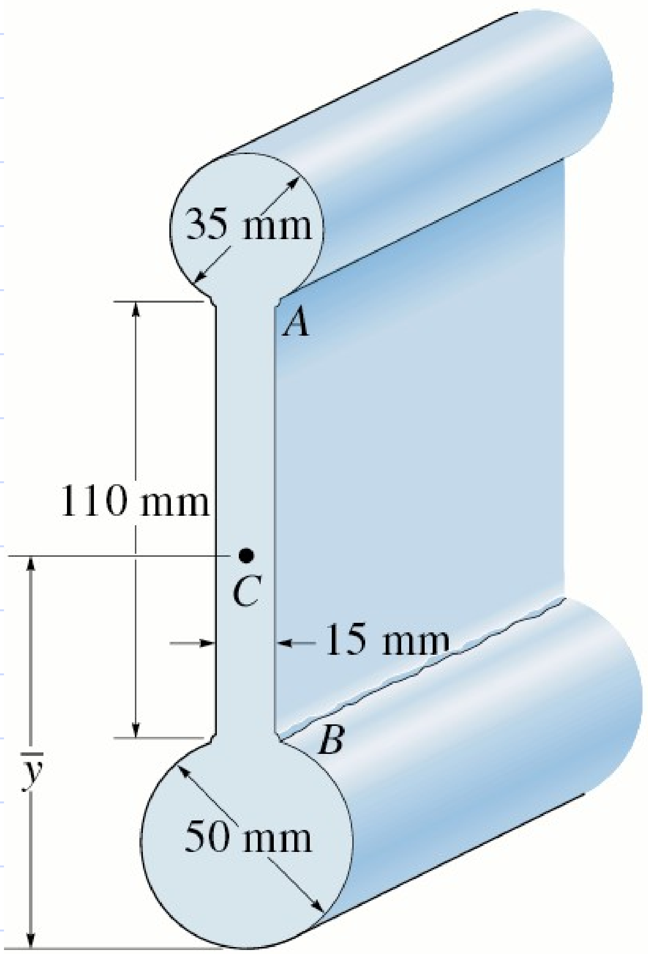


Segment	Area	x	y	xA	yA
1	8	2	1	16	8
2	-0.7854	0.424413	0.424413	-0.33333	-0.33333
3	-1.5	3	1.666667	-4.5	-2.5
	5.714602			11.16667	5.166667
	x=	1.954059			
	y=	0.904117			

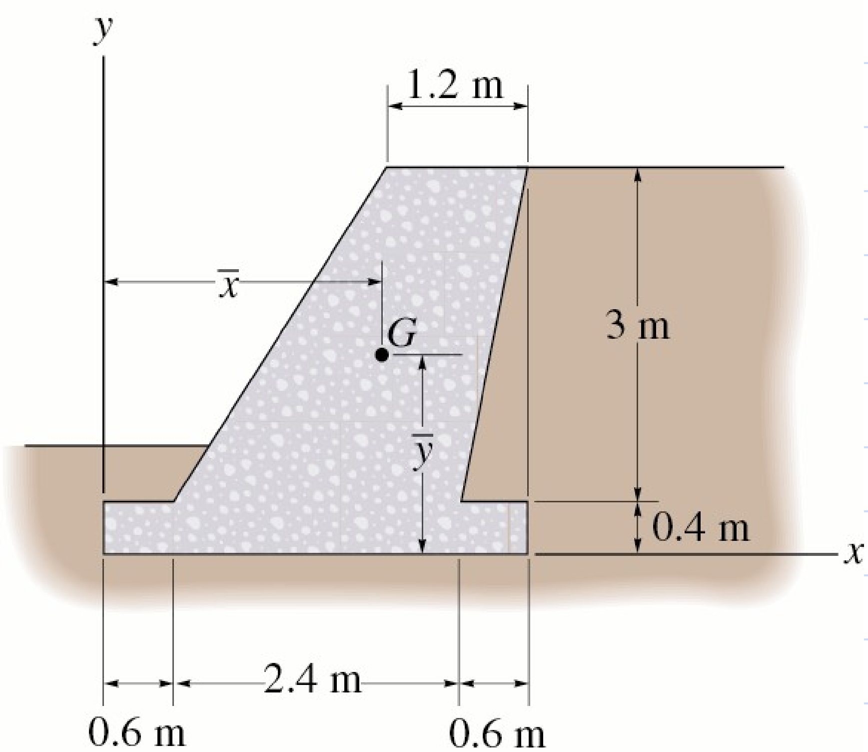
$$A = \frac{1}{2}bh$$

$$\bar{y} = \frac{h}{3}$$

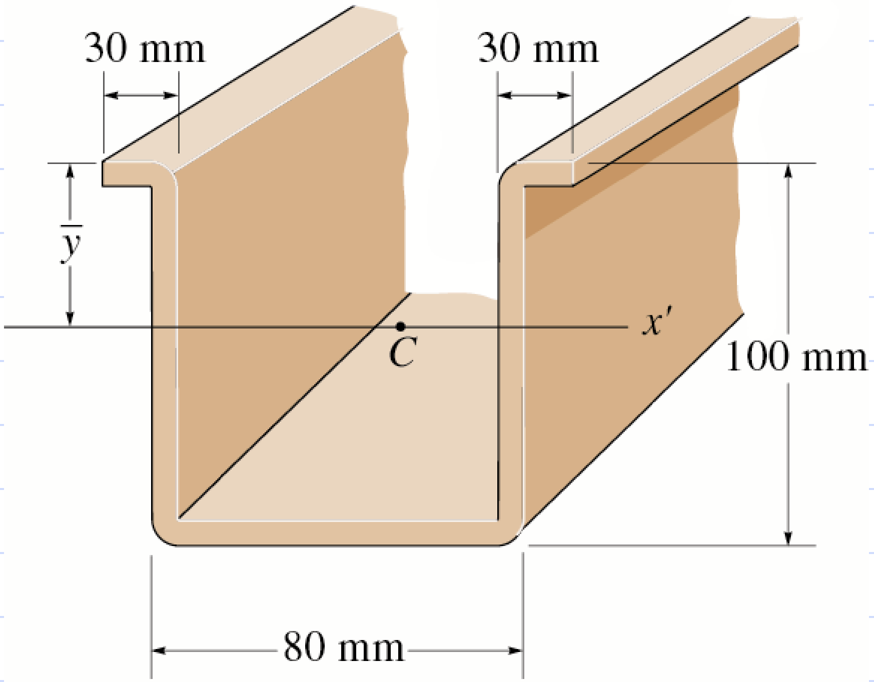




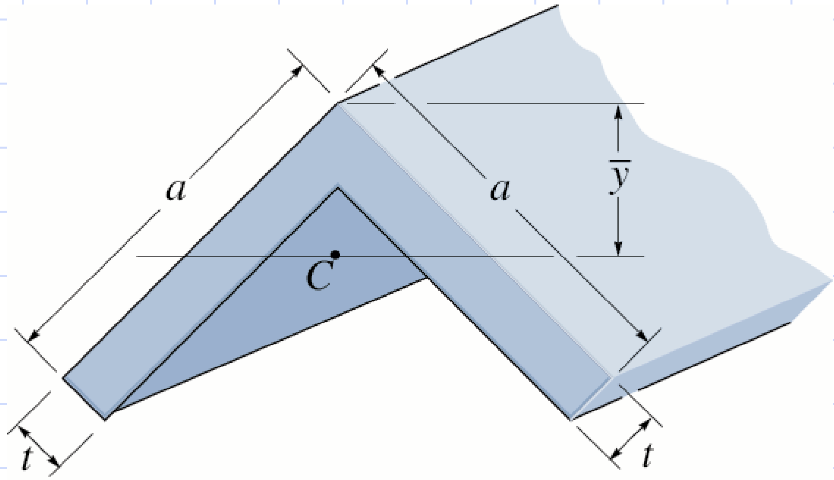
Prob 09.53



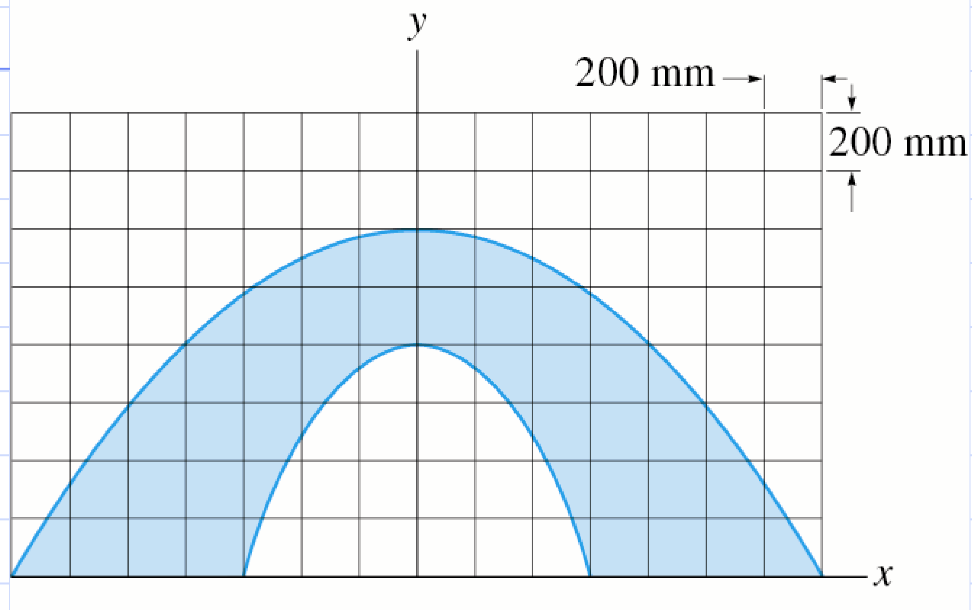
Prob 09.54



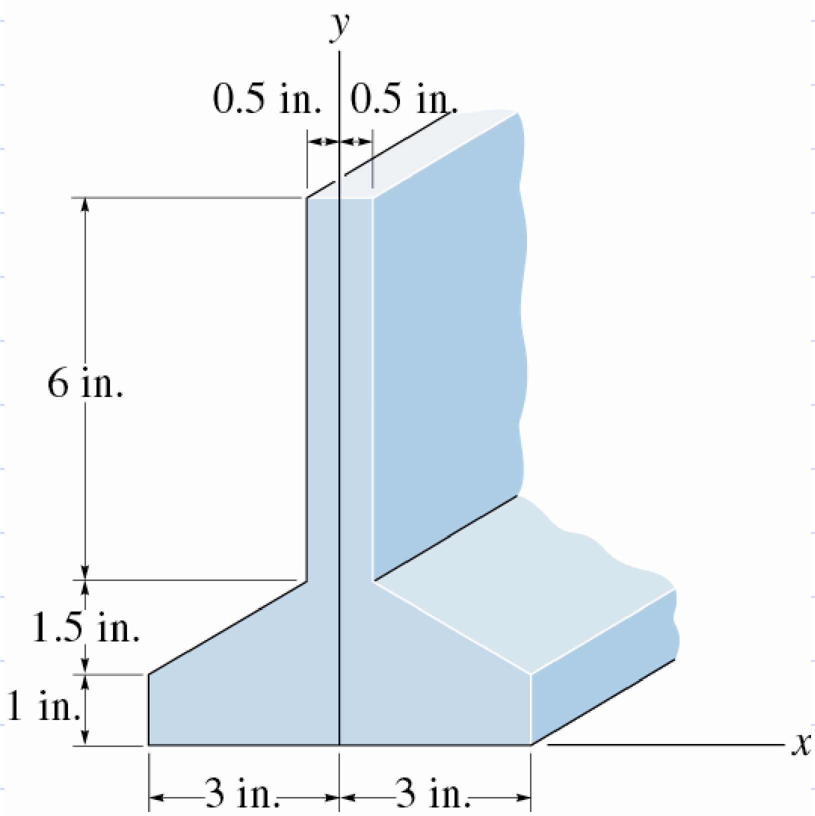
Prob 09.55



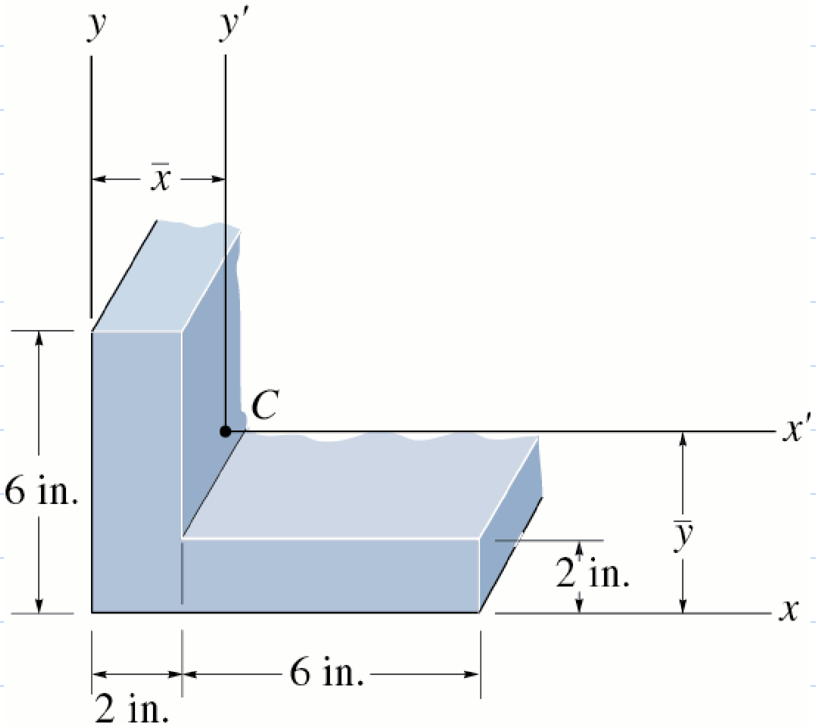
Prob 09.56



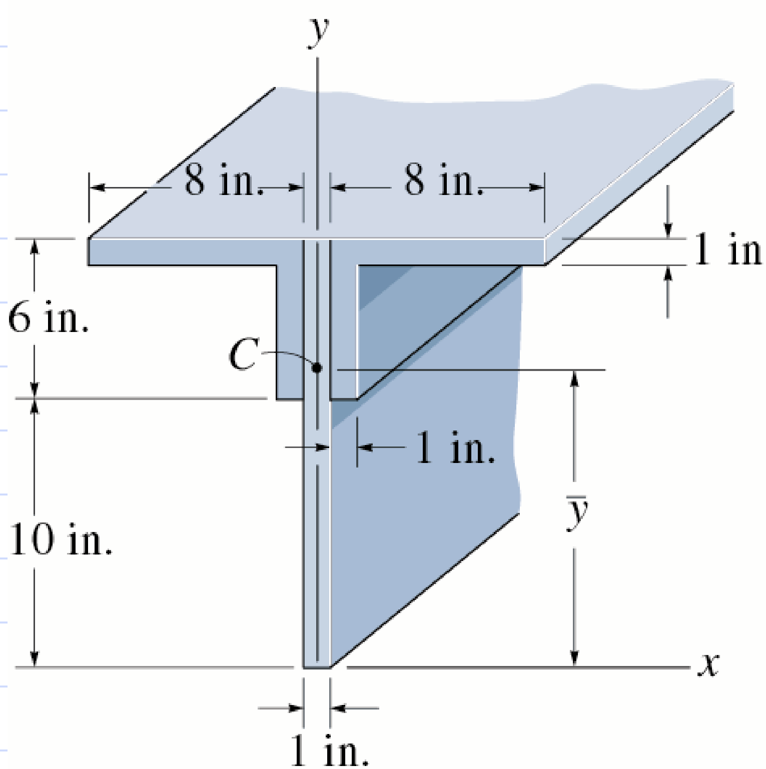
Prob 09.57



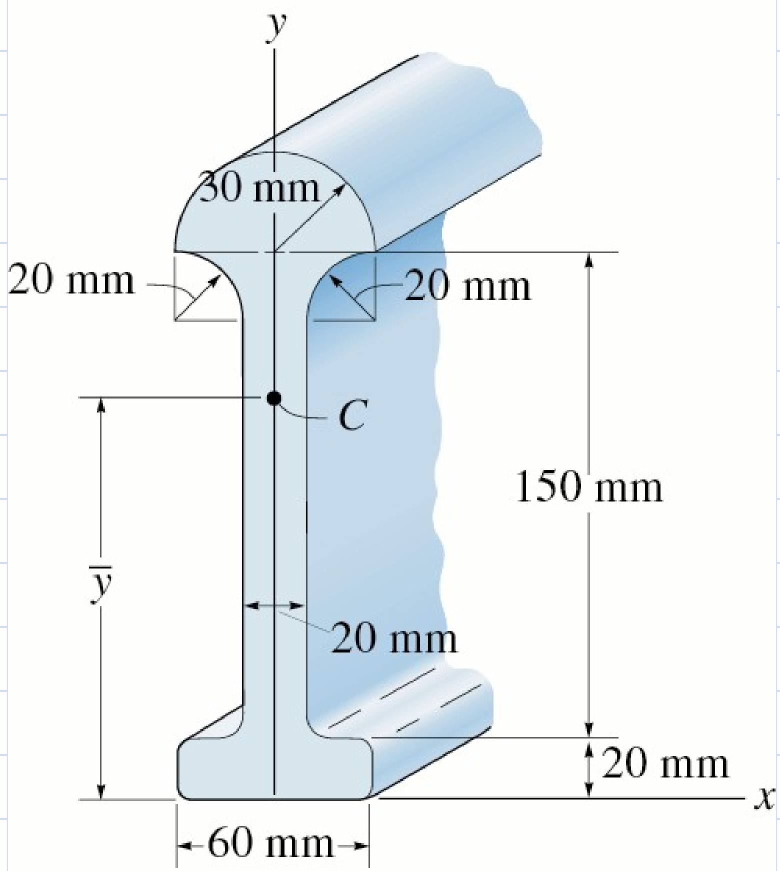
Prob 09.58



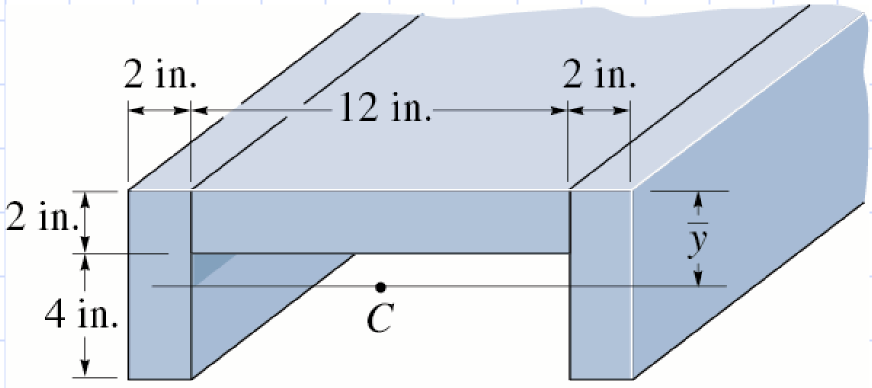
Prob 09.59



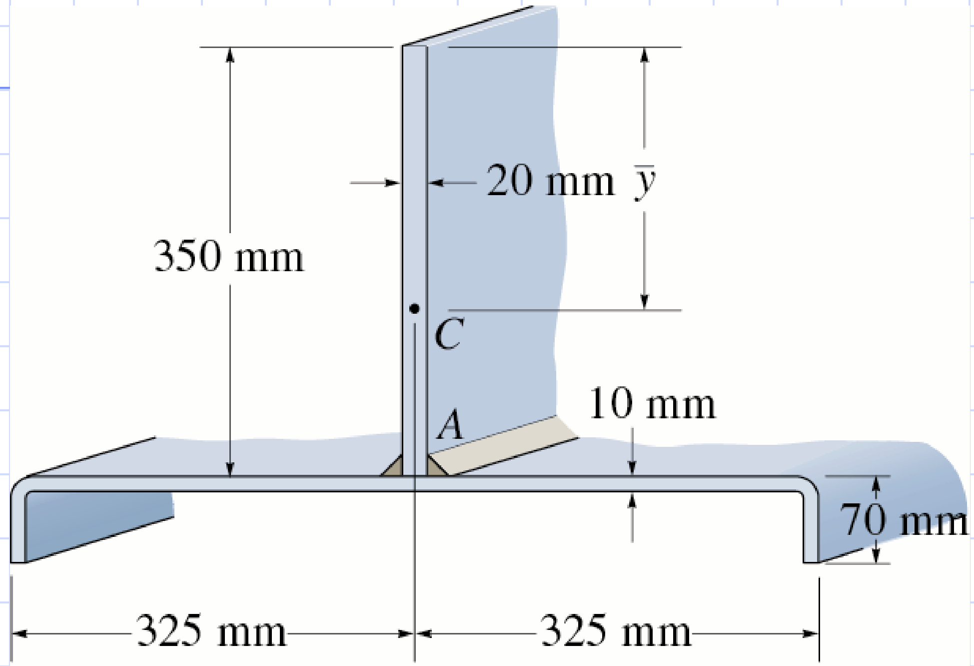
Prob 09.61



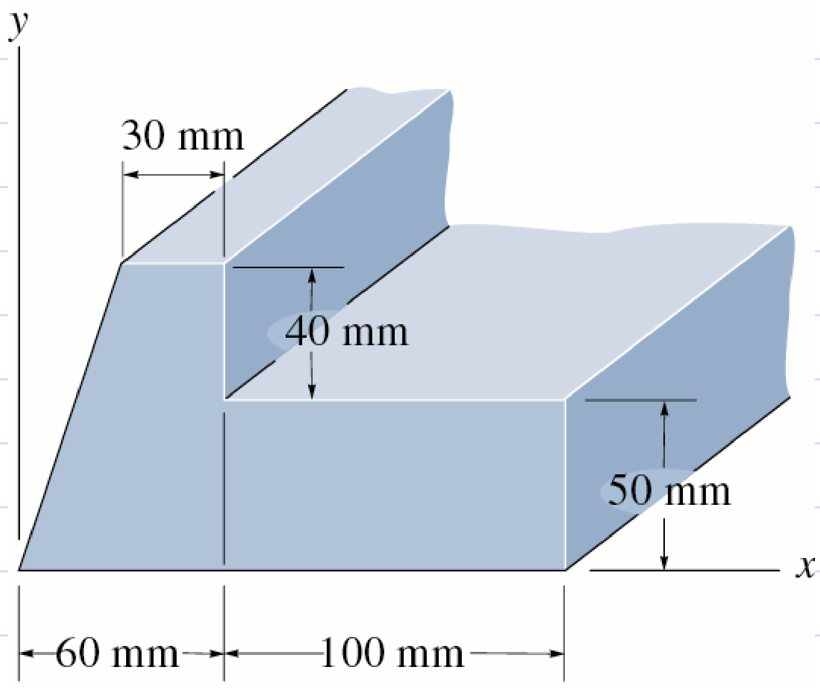
Prob 09.62



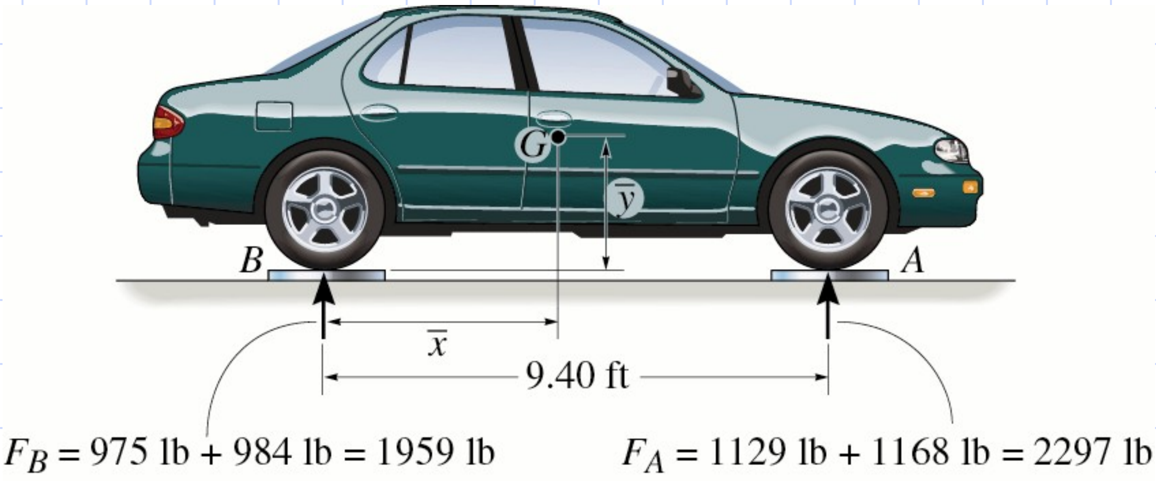
Prob 09.63

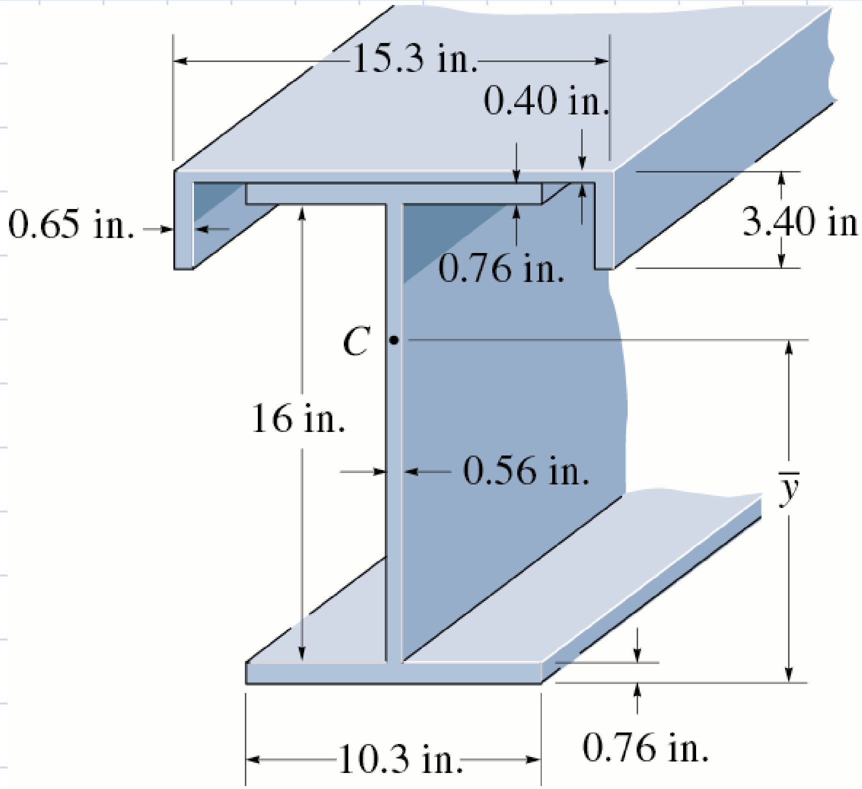


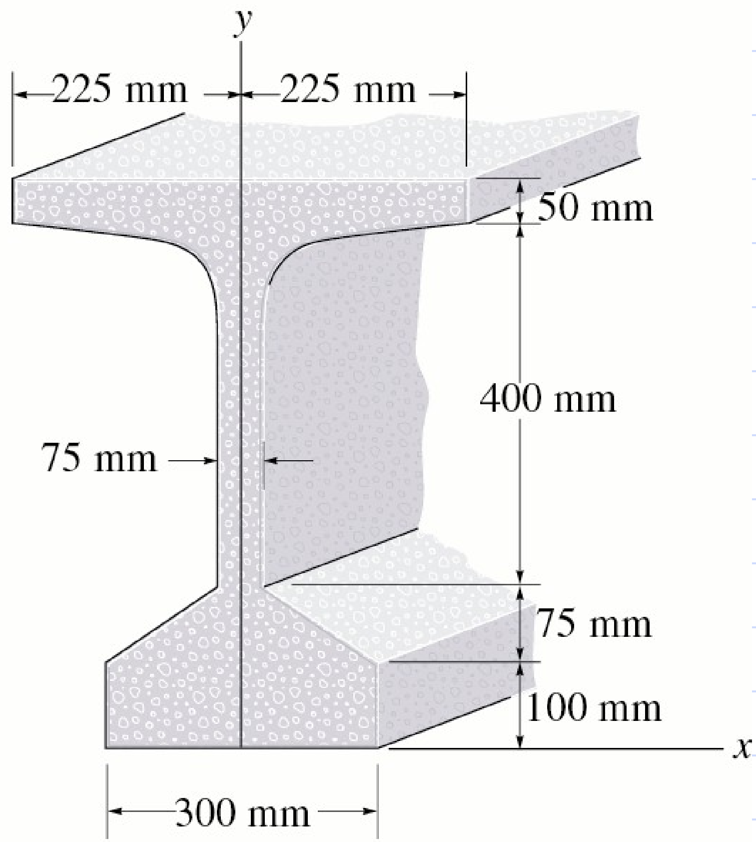
Prob 09.64



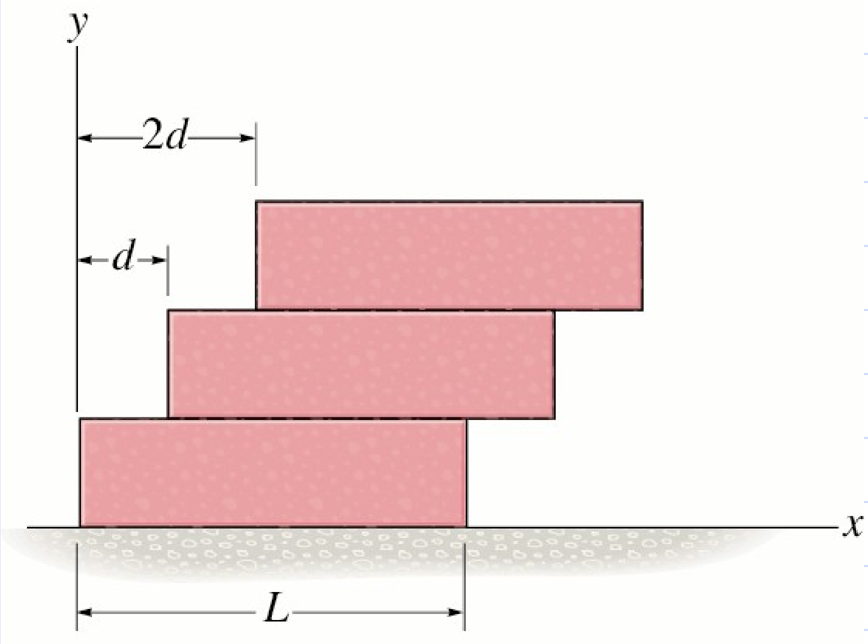
Prob 09.65


G
$\bar{y}$
B
A
$\bar{x}$
9.40 ft
$F_B = 975$ lb + 984 lb = 1959 lb
$F_A = 1129$ lb + 1168 lb = 2297 lb

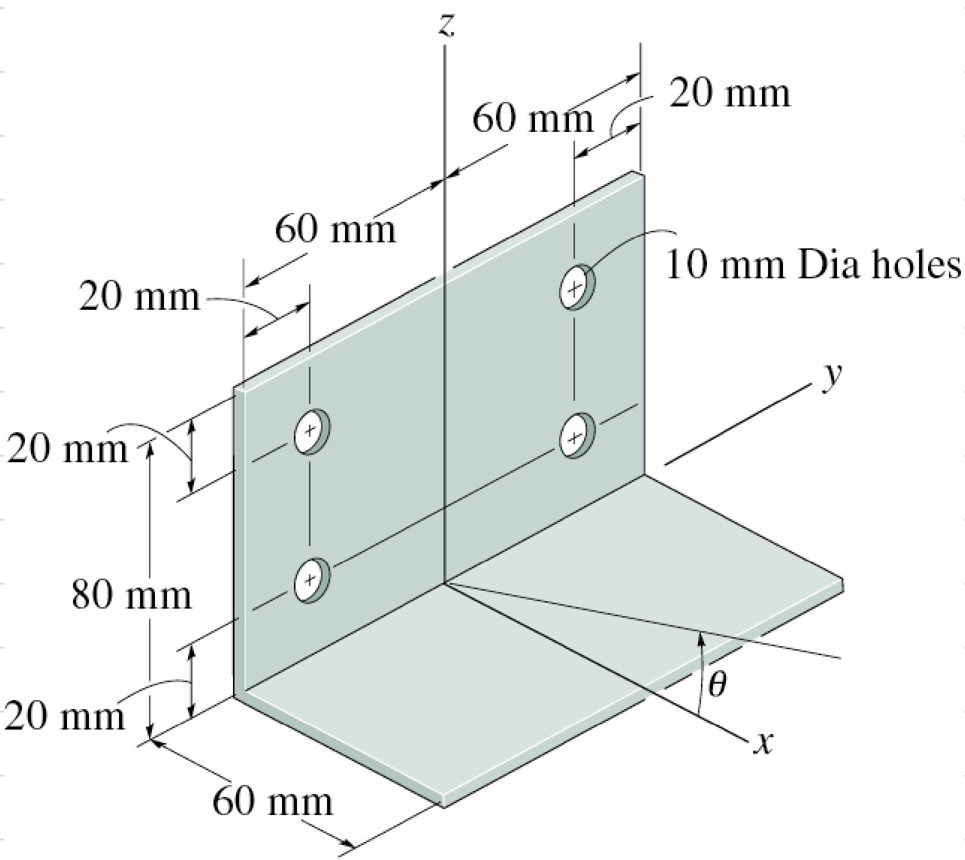

15.3 in.
0.40 in.
0.65 in.
3.40 in.
0.76 in.
C
16 in.
0.56 in.
$\overline{y}$
10.3 in.
0.76 in.
Prob 09.67



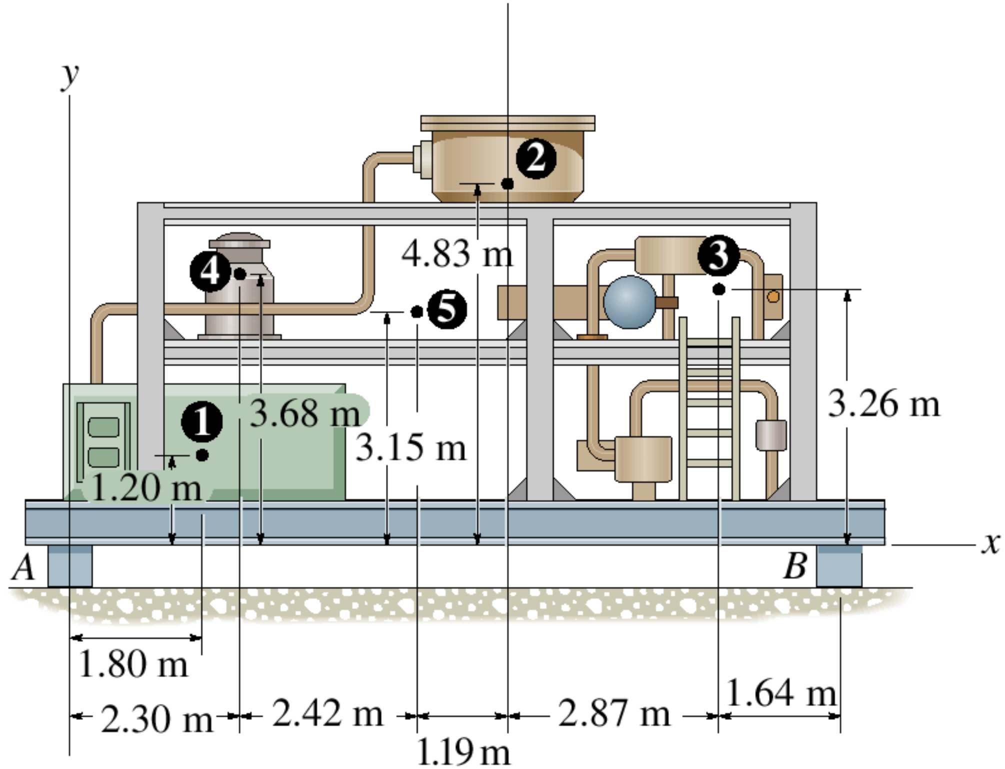
Prob 09.68



Probs 09.69/70



Prob 09.73



1	Instrument panel	230 kg
2	Filter system	183 kg
3	Piping assembly	120 kg
4	Liquid storage	85 kg
5	Structural framework	468 kg

Prob 09.74